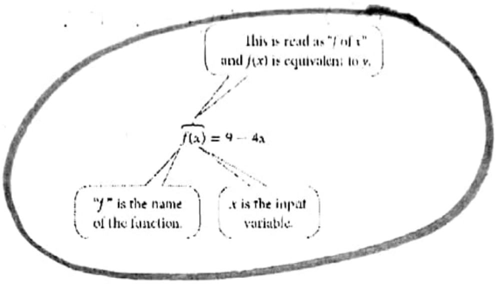
Section 1.6: Functional Notation

Since functions are rules that convert inputs (*x*-values) into outputs (*y*-values), it makes sense that they must have their own notation to indicate what the rule is, what the input is, and what the output is. First, we need to interpret this notation.

FUNCTION NOTATION

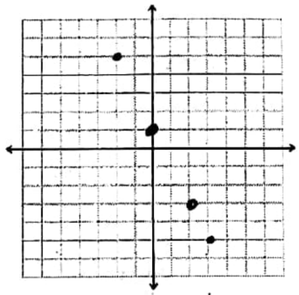
$$y = f\ (x)$$

Output → y Rule → f Input → x



Example 1: $f(x) = 1 - 2x$

input	rule	output	graph
x	$f(x) = 1-2x$	$f(x)$	(x, y)
-2	$f(-2) = 1-2(-2)$	5	$(-2, 5)$
0	$f(0) = 1-2(0)$	1	$(0, 1)$
3	$f(3) = 1-2(3)$	-5	$(3, -5)$



Ex $f(2) = 1-2(2) = -3$

$(2, -3)$

Example 2: For each of the following functions, find the outputs for the given inputs.

$f(x) = 3x + 7$ $g(x) = \dfrac{x-6}{2}$ $h(x) = \sqrt{2x+1}$

$f(2) = 3(2) + 7 = 13$ $g(20) = \dfrac{20-6}{2} = 7$ $h(4) = \sqrt{2(4)+1} = \sqrt{9} = \boxed{3}$

$f(-3) = 3(-3) + 7 = -2$ $g(0) = \dfrac{0-6}{2} = -3$ $h(0) = \sqrt{2(0)+1} = \sqrt{1} = \boxed{1}$

Example 3: Boiling water at 212 degrees Fahrenheit is left in a room that is at 65 degrees Fahrenheit and begins to cool. Temperature readings are taken each hour and are given in the

h (hours)	0	1	2	3	4	5	6	7	8
$T(h)$ (°F)	212	141	104	85	76	70	68	66	65

table below. In this scenario, the temperature, *T*, is a function of the number of hours, *h*. Answer the following. What do these represent in the context of the problem?

a. Evaluate $T(2)$ and $T(6)$.

$T(2) = 104°$
$T(6) = 68°$ } TEMPS.

b. For what value of *h* is $T(h) = 76$?

$h = 4$ TEMP @ 4HRS = 76.

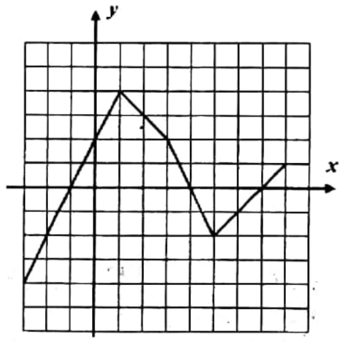
c. Between what two consecutive hours will $T(h) = 100$?

BETWEEN 2 AND 3 HRS.

Example 4: A moving company uses the function $\boxed{C(m) = 20m - 40 \text{ to}}$ calculate the cost, $C(m)$, of moving a families belonging m miles. How much does the company charge a family moving 15 miles to their new home?

$C(15) = 20(15) + 40 = \boxed{\$340}$

Example 5: The function $y = f(x)$ is defined by the graph shown below. It is known as piecewise linear because it is made up of line segments. Answer the following questions based on this graph.



a. Evaluate each of the following:

$f(1) =$ 4 $\quad f(5) =$ -2

$f(-3) =$ -4 $\quad f(0) =$ 2

b. Solve each of the following for all values of the input, x, that make them true.

$f(x) = 0$ $(y = 0)$ $\quad f(x) = 2$ $(y = 2)$

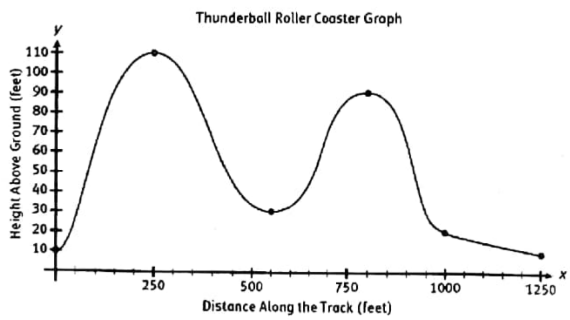
$\{-1, 4, 7\}$ $\quad \{0, 5\}$

c. What is the largest output achieved by the function? At what x-value is it hit?

4 $\quad$ 1

Functions, Domain, and Range with Real-World Applications

Roller coasters are scary and fun to ride. Wooden roller coasters shake and rattle as part of the thrill of the ride. Below is the graph of the heights reached by the cars of the wooden roller coaster, Thunderball, over its first 1250 feet of track. The graph displays a function because each input value has one and only one output value. You can see this visually using the **vertical line test**. Determine the domain and range.



Independent variable is the variable for which input (x) values are substituted in a function.

Independent Variable: DISTANCE ALONG THE TRACK. (FT.)

A **dependent variable** is the variable (y) whose value is determined by the input or value of the independent variable.

Dependent Variable: HT (FT)

Domain: $0 \leq x \leq 1250$

Range: $10 \leq y \leq 110$

Is this data continuous or discrete?

Example 6: Sarah's car holds a maximum of 12 gallons of gas. The function $f(g) = 3.50g$ models the relationship between the cost of gas, $f(g)$, and the number of gallons of gas purchased, g.

g = GALLONS
f(g) = $

a. What is the **most appropriate** domain of the function?

$0 \le g \le 12$

b. What is the **most appropriate** range of the function?

$f(0) = 3.5(0) = 0$
$f(12) = 3.5(12) = 42$

$\boxed{0 \le f(g) \le 42}$

Example 7: A rental company uses the function $f(x) = 150x + 75$ to calculate the cost to rent a beach house x number of nights. The maximum number of nights the beach house can be rented is 30. What is the domain of the function?

INPUT : # OF NIGHTS
OUTPUT : COST ($)

a. $0 \le x \le 30$, where x is a whole number

b. $0 < x < 30$, where x is a whole number

~~**c.**~~ $0 \le x \le 4,575$, where x is a whole number

~~**d.**~~ $0 < x < 4,575$, where x is a whole number

RG VALUES

Example 8: An ice cream shop uses the function $f(p) = 2.50p - 300$ to calculate the amount of profit or loss, $f(p)$, the store makes each day after selling p number of ice cream cones. Which domain is appropriate for the function and shows the ice cream shop making a profit?

PROFIT = REV − COST.
BREAK EVEN: PROFIT = 0
REV = COST.

a. all positive integers

b. all positive rational numbers

c. all integers greater than 120

d. all rational numbers greater than 120

$0 = 2.5p - 300$
$+300 \quad +300$
$\frac{300}{2.5} = \frac{2.5p}{2.5}$ $\quad p = 120$

Example 9: A bucket being filled with water is modeled by the function $y = 2.5x$ where x is the minutes and y is the quarts of water in the bucket. The bucket can hold a maximum of 50 quarts. What is an appropriate domain for this function?

a. $x \ge 0$

b. $x \le 20$

c. $0 \le x \le 20$

d. $0 \le x \le 50$

$y = 2.5x$
$\frac{50}{2.5} = \frac{2.5x}{2.5}$
$20 = x$
0 TO 20 MINUTES.

Functional Notation Practice

1. Given the function f defined by the formula $f(x) = 2x + 1$ find the following:

   a) $f(4)$ b) $f(-5)$ c) $f(0)$ d) $f\left(\frac{1}{2}\right)$

   $2(4)+1=9$ $2(-5)+1=-9$ $2(0)+1=1$ $2(\frac{1}{2})+1=2$

2. Given the function g defined by the formula $g(x) = x^2 - 4$ find the following:

   a) $g(3)$ b) $g(-4)$ c) $g(0)$ d) $g(-2)$

   $3^2-4=5$ $(-4)^2-4=12$ $0^2-4=-4$ $(-2)^2-4=0$

3. If the function $f(x) = 2x - 3$ and $g(x) = \frac{3}{2}x + 1$ then which of the following is a true statement?

   a) $f(0) > g(0)$ ✗
   b) $f(2) = g(2)$ ✗
   c) $f(8) = g(8)$ ✓
   d) $g(4) < f(4)$ ✗

x	f(x)
0	-3
2	1
8	13
4	5

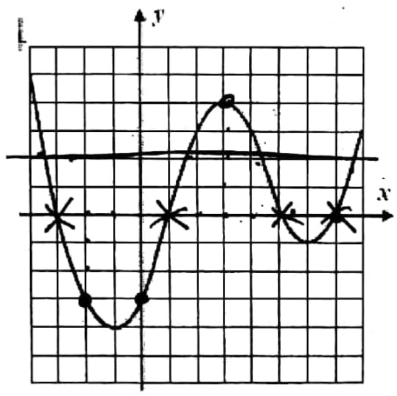
x	g(x)
0	1
2	4
8	13
4	7

4. Based on the graph of the function $y = g(x)$ shown below, answer the following questions.

   a) Evaluate each of the following. Illustrate with a point on the graph.

   $g(-2) =$ -3 $g(0) =$ -3

   $g(3) =$ 4 $g(7) =$ 0



   b) What values of x solve the equation $g(x) = 0$?
   (These are called **zeroes of the function**.)

   $\{-3, 1, 5, 7\}$

   c) How many values of x solve the equation $g(x) = 2$? How can you illustrate your answer on the graph? Remember, we are not looking for the exact x-values, only **how many solutions**.

   4   DRAW $y=4$

5. A pack of pencils cost \$0.75. If n number of packs are purchased, then the total purchase price is represented by the function $t(n) = 0.75n$.

   a) Explain why t is a function.   1 OUTPUT FOR EACH INPUT.

   b) What is a reasonable domain and range for the function t?

   D: $\{0, 1, \ldots\}$

   R: $\{.75, 1.50, \ldots\}$

6. Suppose f is a function.

a) If $10 = f(-4)$, give the coordinates of a point on the graph of f.

← y ← x

(−4, 10)

b) If 6 is a solution of the equation $f(w) = 1$, give a point on the graph of f.

↑ x ↑ y

(6, 1)

7. You placed a yam in the oven and, after 45 minutes, you take it out. Let f be the function that assigns to each minute after you placed the yam in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the following to explain what it means in everyday language.

← TIME ← TEMP

a) $f(0) = 65$ AT START, TEMP = 65°F.

b) $f(5) < f(10)$ HOTTER @ 10 MIN THAN 5 MIN.

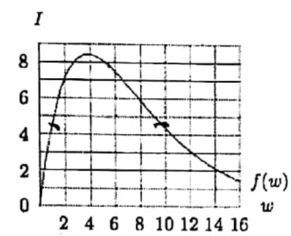
c) $f(40) = f(45)$ SAME TEMP @ 40 MIN + 45 MIN.

d) $f(45) > f(60)$ COOLER @ 60 MIN THAN 45 MIN.

8. An epidemic of influenza spreads through a city. The figure below is the graph of $I = f(w)$, where I is the number of individuals (in thousands) infected w weeks after the epidemic begins.

a) Estimate $f(2)$ and explain its meaning in terms of the epidemic.

f(2) = 7 7000 PEOPLE INFECTED.



b) Approximately how many people were infected at the height of the epidemic? When did that occur? Write your answer in the form $f(a) = b$.

8500

f(4) = 8500

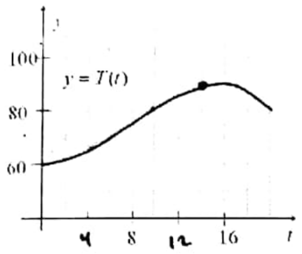
c) For approximately which w is $f(w) = 4.5$; explain what the estimates mean in terms of the epidemic.

f(1) = 4.5 AT 1 WK and 10 WKS

f(10) = 4.5 4500 PEOPLE HAVE DISEASE.

d) An equation for the function used to plot the image above is $f(w) = 6w(1.3)^{-w}$. Use the graph to estimate the solution of the inequality $6w(1.3)^{-w} \geq 6$. Explain what the solution means in terms of the epidemic.

e) The figure shows the graph of T, the temperature (in degrees Fahrenheit) over one particular 20-hour period, in Santa Elena as a function of time t.



a. Estimate $T(14)$. 90° 89°

b. If $t = 0$ corresponds to midnight, interpret what we mean by $T(14)$ in words. AT 2 PM TEMP IS 89°

c. Estimate the highest temperature during this period from the graph. 90°

d. When was the temperature decreasing? $t > 16$

e. If Anya wants to go for a two-hour hike and return before the temperature gets over 80 degrees, when should she leave?

$f(10) = 80$

BEFORE 8 AM.

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