

SECTION 3.2

Radical Lesson

\sqrt{n} - Radical $\sqrt{\quad}$ - Radical Symbol n - Radicand

Simplifying Radicals (Square Roots):

- 1) Find the largest perfect square factor of the radicand.
- 2) Factor the radicand using the perfect square factor.
- 3) $\sqrt{\text{Perfect Square}}$ comes outside the radical, the other factor (non-perfect square) stays inside the radical.

Ex: Simplify. (Exact Roots – no decimal approximations)

$$1) \sqrt{18} = 3\sqrt{2}$$

$\begin{array}{c} \wedge \\ 2 \quad 9 \\ \wedge \\ 3 \quad 3 \end{array}$

$$2) \sqrt{24} = 2\sqrt{6}$$

$\begin{array}{c} \wedge \\ 2 \quad 12 \\ \wedge \\ 2 \quad 6 \\ \wedge \\ 2 \quad 3 \end{array}$

$$3) \sqrt{147} = 7\sqrt{3}$$

$\begin{array}{c} \wedge \\ 3 \quad 49 \\ \wedge \\ 7 \quad 7 \end{array}$

$$4) \sqrt{605} = 11\sqrt{5}$$

$\begin{array}{c} \wedge \\ 5 \quad 125 \\ \wedge \\ 11 \quad 25 \\ \wedge \\ 5 \quad 5 \end{array}$

$$5) \sqrt{72} = 6\sqrt{2}$$

$\begin{array}{c} \wedge \\ 2 \quad 36 \\ \wedge \\ 6 \quad 6 \end{array}$

$$6) \sqrt{84} = 2\sqrt{21}$$

$\begin{array}{c} \wedge \\ 2 \quad 42 \\ \wedge \\ 2 \quad 21 \\ \wedge \\ 3 \quad 7 \end{array}$

$$7) \sqrt{128} = 8\sqrt{2}$$

$\begin{array}{c} \wedge \\ 2 \quad 64 \\ \wedge \\ 8 \quad 8 \end{array}$

$$8) \sqrt{49x^2} = 7x$$

$\begin{array}{c} \wedge \quad \wedge \\ 7 \quad 7 \quad x \quad x \end{array}$

$$9) \sqrt{64x^2y^3} = 8x\sqrt{y}$$

$\begin{array}{c} \wedge \quad \wedge \quad \wedge \\ 8 \quad 8 \quad x \quad x \quad y \quad y \quad y \end{array}$

$$10) \sqrt{125a^2b} = 5a\sqrt{5b}$$

$\begin{array}{c} \wedge \\ 5 \quad 25 \\ \wedge \\ 5 \quad 5 \end{array}$

$$11) \sqrt{98c^3d^4} = 7cd^2\sqrt{2c}$$

$\begin{array}{c} \wedge \\ 2 \quad 49 \\ \wedge \\ 7 \quad 7 \end{array}$

Multiplying Radicals: $a\sqrt{b} \cdot c\sqrt{d} = ac\sqrt{bd}$

Ex: Simplify.

$$1) 3\sqrt{5} \cdot \sqrt{2}$$

$$= 3\sqrt{10}$$

$$2) 2\sqrt{14} \cdot \sqrt{21}$$

$\begin{array}{c} \wedge \quad \wedge \\ 2 \quad 7 \quad 3 \quad 7 \end{array}$

$$2\sqrt{2 \cdot 7 \cdot 3 \cdot 7} =$$

$$2 \cdot 7 \sqrt{2 \cdot 3} =$$

$$14\sqrt{6}$$

$$3) 2\sqrt{10} \cdot 5\sqrt{5} =$$

$$10\sqrt{50} = 10 \cdot 5\sqrt{2} =$$

$$50\sqrt{2}$$

$\begin{array}{c} \wedge \\ 2 \quad 25 \\ \wedge \\ 5 \quad 5 \end{array}$

(9)

A Radical Expression is simplified when:

- 1) The radicand does not contain any perfect square factors.
- 2) The radicand is not a fraction.
- 3) No radicals in the denominator. (Rationalize the Denominator)

Dividing Radicals: $\frac{a\sqrt{b}}{c\sqrt{d}} = \frac{a}{c}\sqrt{\frac{b}{d}}$

Ex: Simplify. $\frac{6\sqrt{15}}{2\sqrt{5}} = \frac{6}{2} \cdot \sqrt{\frac{15}{5}} = 3\sqrt{3}$

Rationalize the Denominator: Multiply the numerator and denominator by a quantity so the denominator is an exact root.

$$\frac{\sqrt{a}}{\sqrt{b}} \cdot 1 = \frac{\sqrt{a}}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{\sqrt{b^2}} = \frac{\sqrt{ab}}{b}$$

Ex: Simplify.

1) $\frac{\sqrt{11}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{66}}{6}$

2) $\frac{2\sqrt{7}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{35}}{5}$

3) $\frac{3\sqrt{5}}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}} = \frac{3\sqrt{40}}{8} =$

$$\frac{3 \cdot 2\sqrt{10}}{8} = \frac{6\sqrt{10}}{8} = \frac{3\sqrt{10}}{4}$$

Operations with Radical Expressions: Radical expressions in which the radicands are alike can be added or subtracted in the same way that monomials are added or subtracted.

Monomials

$$4x + 5x = 9x$$

Radical Expressions

$$4\sqrt{5} + 5\sqrt{5} = 9\sqrt{5}$$

10

Ex: Simplify each expression.

1) $6\sqrt{7} + 5\sqrt{7} - 3\sqrt{7}$

$8\sqrt{7}$

2) $5\sqrt{6} + 3\sqrt{7} + 4\sqrt{7} - 2\sqrt{6}$

$3\sqrt{6} + 7\sqrt{7}$



If the radicals in a radical expression are not in simplest form, simplify each radical first. Then look to see if there are like radicals.

3) $4\sqrt{27} + 5\sqrt{12} + 8\sqrt{75}$

$\begin{matrix} \wedge & \wedge & \wedge \\ 3\ 9 & 3\ 4 & 3\ 25 \\ \wedge & \wedge & \wedge \\ 3\ 3 & 2\ 2 & 5\ 5 \end{matrix}$

$4 \cdot 3\sqrt{3} + 2 \cdot 5\sqrt{3} + 5\sqrt{3}$

$12\sqrt{3} + 10\sqrt{3} + 5\sqrt{3}$

$27\sqrt{3}$

4) $3\sqrt{45} + 4\sqrt{80} - 2\sqrt{125}$

$\begin{matrix} \wedge & \wedge & \wedge \\ 3\ 15 & 2\ 40 & 5\ 5 \\ \wedge & \wedge & \wedge \\ 3\ 5 & 2\ 20 & 5\ 5 \\ & \wedge & \\ & 2\ 10 & \\ & \wedge & \\ & 2\ 5 & \end{matrix}$

$9\sqrt{5} + 16\sqrt{5} - 10\sqrt{5} =$

$15\sqrt{5}$

Radical Practice

Directions: Simplify. (Exact Roots – no decimal approximations)

1) $\sqrt{45}$
 $3\sqrt{5}$

2) $\sqrt{48}$
 $4\sqrt{3}$

3) $\sqrt{216}$
 $6\sqrt{6}$

4) $\sqrt{12}$
 $2\sqrt{3}$

5) $8\sqrt{27}$
 $24\sqrt{3}$

6) $10\sqrt{96}$
 $40\sqrt{6}$

7) $2\sqrt{200}$
 $20\sqrt{2}$

8) $\sqrt{50x^2}$
 $5x\sqrt{2}$

9) $\sqrt{144x^3y}$
 $12x\sqrt{xy}$

10) $3\sqrt{12} \cdot \sqrt{6}$
 $18\sqrt{2}$

11) $2\sqrt{5} \cdot 3\sqrt{15}$
 $30\sqrt{3}$

12) $\frac{\sqrt{4}}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{12}}{15} = \frac{2\sqrt{3}}{15}$

13) $\frac{\sqrt{8}}{\sqrt{100}}$
 $\frac{2\sqrt{2}}{10} = \frac{\sqrt{2}}{5}$

14) $\frac{\sqrt{15}}{2\sqrt{20}} \cdot \frac{\sqrt{20}}{\sqrt{20}}$
 $\frac{\sqrt{300}}{40} = \frac{10\sqrt{3}}{40} = \frac{\sqrt{3}}{4}$

15) $\frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{3}$

16) $\frac{2\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{14}}{7}$

17) $-2\sqrt{3} + 3\sqrt{27}$
 $-2\sqrt{3} + 9\sqrt{3}$
 $7\sqrt{3}$

18) $-\sqrt{27} - 3\sqrt{45} - \sqrt{20} + 2\sqrt{75}$
 $-\underline{3\sqrt{3}} - \underline{9\sqrt{5}} - \underline{2\sqrt{5}} + \underline{10\sqrt{3}}$
 $7\sqrt{3} - 11\sqrt{5}$

(12)

Simplifying Radical Expressions

Simplify.

1) $\sqrt{125n}$ $5\sqrt{5n}$

2) $\sqrt{216v}$ $6\sqrt{6v}$

3) $\sqrt{512k^2}$ $16k\sqrt{2}$

4) $\sqrt{512m^3}$ $16m\sqrt{2m}$

5) $\sqrt{216k^4}$ $6k^2\sqrt{6}$

6) $\sqrt{100v^3}$ $10v\sqrt{v}$

7) $\sqrt{80p^3}$ $4p\sqrt{5p}$

8) $\sqrt{45p^2}$ $3p\sqrt{5}$

9) $\sqrt{147m^3n^3}$
 $7mn\sqrt{3mn}$

10) $\sqrt{200m^4n}$ $10m^2\sqrt{2n}$

11) $\sqrt{75x^2y}$
 $5x\sqrt{3y}$

12) $\sqrt{64m^3n^3}$ $8mn\sqrt{mn}$

13) $\sqrt{16u^4v^3}$
 $4u^2\sqrt{v}$

14) $\sqrt{28x^3y^3}$ $2xy\sqrt{7xy}$

Adding and Subtracting Radical Expressions

Simplify.

1) $3\sqrt{6} - 4\sqrt{6}$
 $-\sqrt{6}$

2) $-3\sqrt{7} + 4\sqrt{7}$
 $\sqrt{7}$

3) $-11\sqrt{21} - 11\sqrt{21}$
 $-22\sqrt{21}$

4) $-9\sqrt{15} + 10\sqrt{15}$
 $\sqrt{15}$

5) $-10\sqrt{7} + 12\sqrt{7}$
 $2\sqrt{7}$

6) $-3\sqrt{17} - 4\sqrt{17}$
 $-7\sqrt{17}$

7) $-10\sqrt{11} - 11\sqrt{11}$
 $-21\sqrt{11}$

8) $-2\sqrt{3} + 3\sqrt{27}$
 $7\sqrt{3}$

9) $2\sqrt{6} - 2\sqrt{24}$
 $-2\sqrt{6}$

10) $2\sqrt{6} + 3\sqrt{54}$
 $11\sqrt{6}$

11) $-\sqrt{12} + 3\sqrt{3}$
 $\sqrt{3}$

12) $3\sqrt{3} - \sqrt{27}$
 0

Multiplying Radical Expressions

Simplify.

1) $3\sqrt{12} \cdot \sqrt{6}$

$18\sqrt{2}$

2) $\sqrt{5} \cdot \sqrt{10}$

$5\sqrt{2}$

3) $\sqrt{6} \cdot \sqrt{6}$

6

4) $\sqrt{5} \cdot -4\sqrt{20}$

-40

5) $-4\sqrt{15} \cdot -\sqrt{3}$

$12\sqrt{5}$

6) $\sqrt{20x^2} \cdot \sqrt{20x}$

$20x\sqrt{x}$

7) $\sqrt{15n^2} \cdot \sqrt{10n^3}$

$5n^2\sqrt{6n}$

8) $\sqrt{18a^2} \cdot 4\sqrt{3a^2}$

$12a^2\sqrt{6}$

9) $-3\sqrt{7r^3} \cdot 6\sqrt{7r^2}$

$-126r^2\sqrt{r}$

10) $-4\sqrt{28x} \cdot \sqrt{7x^3}$

$-56x^2$

11) $\sqrt{3}(5 + \sqrt{3})$

$5\sqrt{3} + 3$

12) $2\sqrt{5}(\sqrt{6} + 2)$

$2\sqrt{30} + 4\sqrt{5}$

13) $-3\sqrt{3}(2 + \sqrt{6})$

$-6\sqrt{3} - 9\sqrt{2}$

14) $\sqrt{3}(-5\sqrt{10} + \sqrt{6})$

$-5\sqrt{30} + 3\sqrt{2}$