Section 2.3 Average Rates of Change MAYO F2017

NC.M1.F-IF.6 Interpret functions in applications in terms of the context. Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

Review. Complete the following. Show work!

1. If f(x) = 3x + 2, find:

a. f(2)

$f(2) = 3(2) + 2$
$= 6 + 2 = 8$

b. f(-1)

$f(-1) = 3(-1) + 2$
$= -3 + 2 = -1$

c. x if f(x) = 29

$29 = 3x + 2$
$-2 \quad\quad -2$
$\frac{27}{3} = \frac{3x}{3}$
$x = 9$

2. Compute the slope of the line perpendicular to the line that passes through the points (-2, 5) and (3, -5).

$m = \frac{5-(-5)}{-2-3} = \frac{10}{-5} = -2$

$m_\perp = \frac{1}{2}$

In previous lessons, we have computed the rate of change for *linear functions* by computing slope.

Slope = $\frac{RISE}{RUN} = \frac{y_2 - y_1}{x_2 - x_1}$

We should know that the slope we computed can be thought of two ways:

1. HOW "STEEP" THE GRAPH OF A LINE IS.

2. THE RATE OF CHANGE OF THE FUNCTION REPRESENTED BY THE LINE.

It turns out that this concept of rate of change can actually apply to any type of function, not just linear functions.

A more sophisticated formula for an *Average Rate of Change* for any function f(x) over an interval a ≤ x ≤ b is:

Average Rate of Change = $\frac{f(b) - f(a)}{b - a}$ $\left(\frac{RISE}{RUN}\right)$

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Example 1 (Numerical). The table below shows the weight of a type of plankton after several weeks.

Time (weeks)	Weight (ounces)
8	0.04
9	0.07
10	0.14
11	0.25
12	0.49

2 0.10

2 .35

a. What is the average rate of change in weight of the plankton from week 8 to week 10?

(8, 0.04) (10, 0.14)

AVG RATE OF $\Delta = \frac{0.14 - 0.04}{10 - 8} = \frac{0.10}{2} = 0.05$ OZ/WK

b. What is the average rate of change in weight of the plankton from week 10 to week 12?

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(10, 0.14) (12, 0.49)

AVG RATE OF $\Delta = \frac{0.49 - 0.14}{12 - 10} = \frac{.35}{2} = 0.175 \frac{OZ}{WK}$

c. What is the average rate of change in weight of the plankton from week 8 to week 12?

(8, 0.04) (12, 0.49)

AVG RATE OF $\Delta = \frac{0.49 - 0.04}{12 - 8} = \frac{0.45}{4} = 0.1125$ OZ/WK

Application 1. The table below shows the height of an object, h(t), in feet as a function of time, t, in seconds.

t (sec)	0	1	2	3	4	5
h(t) (ft)	8	9	12	17	24	33

a. Is the average rate of change of h(t) constant? Why?

NO. GOES UP 1, THEN 3, THEN 5, 7, 11.

b. What is the average rate of change of h(t) over the interval 0 to 5 seconds?

$\frac{h(5) - h(0)}{5 - 0} = \frac{33 - 8}{5 - 0} = \frac{25}{5} = \boxed{5 \text{ ft/sec}}$

c. What is the average rate of change of h(t) over the interval 2 to 3 seconds?

$\frac{h(3) - h(2)}{3 - 2} = \frac{17 - 12}{3 - 2} = \frac{5}{1} = \boxed{5 \text{ ft/sec}}$

Example 2 (Algebraic): Suppose we have the function $f(x) = x^2 + 1$.

a. What is the average rate of change of f(x) on the interval from x = 0 to x = 1?

$$\frac{f(1)-f(0)}{1-0} = \frac{2-1}{1-0} = \frac{1}{1} = 1$$

$f(0) = 0^2+1 = 1$
$f(1) = 1^2+1 = 2$

b. What is the average rate of change of f(x) on the interval from x = 1 to x = 2?

$$\frac{f(2)-f(1)}{2-1} = \frac{5-2}{2-1} = \frac{3}{1} = 3$$

$f(1) = 2$
$f(2) = 2^2+1 = 5$

c. What is the average rate of change of f(x) on the interval from x = 2 to x = 3?

$$\frac{f(3)-f(2)}{3-2} = \frac{10-5}{3-2} = \frac{5}{1} = 5$$

$f(2) = 5$
$f(3) = 3^2+1 = 10$

d. What is happening to the average rates of change as the intervals changed?

AVG RATE OF CHANGE IS INCREASING.

Application 2. Find the average rate of change of each of the following functions over the interval $1 \le x \le 5$.

a. f(x) = 3x – 7

$f(1) = 3(1)-7 = 3-7 = -4$
$f(5) = 3(5)-7 = 15-7 = 8$

$$\frac{f(5)-f(1)}{5-1} = \frac{8-4}{5-1} = \frac{4}{4} = 1$$

b. g(x) = x² + 2x – 5

$g(1) = 1^2+2(1)-5 = 1+2-5 = -2$
$g(5) = 5^2+2(5)-5 = 25+10-5 = 30$

$$\frac{g(5)-g(1)}{5-1} = \frac{30-2}{5-1} = \frac{28}{4} = 7$$

c. h(x) = 3(2)ˣ

$h(1) = 3(2)^1 = 6 \quad h(5) = 3(2)^5 = 32$

$$\frac{h(5)-h(1)}{5-1} = \frac{32-6}{4} = \frac{26}{4} = \frac{13}{2} = 6.5$$

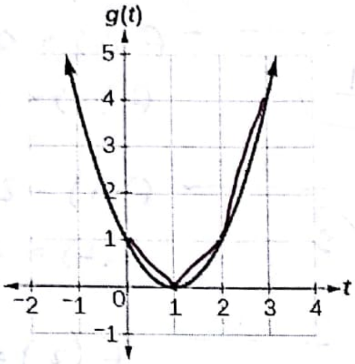
d. k(x) = -2x + 5

$k(1) = -2(1)+5 = 3 \quad k(5) = -2(5)+5 = -5$

$$\frac{k(5)-k(1)}{5-1} = \frac{-5-3}{5-1} = \frac{-8}{4} = -2$$

(DISCUSS COMPARISON.)

Example 3 (Graphical):



DISCUSS ↑ AND ↓; CONCAVITY, etc.

Given the function g(t) whose graph is shown, find the average rate of change on the following intervals

a. from t = 0 to t = 1.

(0,1) (1,0)

$\frac{1-0}{0-1} = -1$

b. from t = 1 to t = 2.

(1,0) (2,1)

$\frac{1-0}{2-1} = 1$

(DISCUSS SLOPES FROM GRAPH.)

c. from t = 2 to t = 3.

(2,1) (3,4)

$\frac{4-1}{3-2} = \frac{3}{1} = 3$

Application 3.

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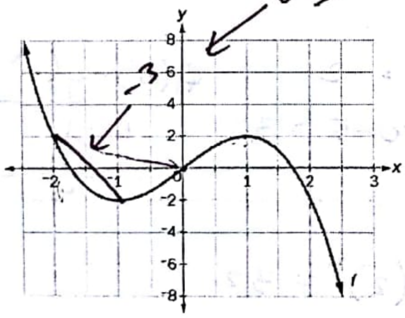


Given the function f(x) whose graph is shown, find the average rate of change on the following intervals:

a. x = -2 to x = -1 (-2,2) (-1,-2)

$\frac{-2-2}{-1+(+2)} = \frac{-4}{1} = -4$

4 1

b. x = - 2 to x = 0

(-2,2) (0,0) $\frac{0-2}{0-(-2)} = \frac{-2}{2} = -1$

c. x = -1 to x = 1

(-1,-2) (1,2) $\frac{2-(-2)}{1-(-1)} = \frac{4}{2} = 2$

d. x = 1 to x = 2.

(1,2) (2,-2) $\frac{-2-2}{2-1} = -4$

Practice.

1. Find the average rate of change for each of the following functions on the interval $2 \le x \le 5$. Show work.

a. $f(x) = 3x - 5$

$f(5) = 3(5) - 5 = 15 - 5 = 10$
$f(2) = 3(2) - 5 = 6 - 5 = 1$

$\frac{10-1}{5-2} = \frac{9}{3} = 3$

b. $g(x) = 3x^2 + 2x - 4$

$g(5) = 3(5)^2 + 2(5) - 4 = 75 + 10 - 4 = 81$

$g(2) = 3(2)^2 + 2(2) - 4 = 12 + 4 - 4 = 12$

$\frac{81-12}{5-2} = \frac{69}{3} = 23$

c. $h(x) = 2(x - 2) + 5$

$h(5) = 2(5-2) + 5 = 6 + 5 = 11$
$h(2) = 2(2-2) + 5 = 0 + 5 = 5$

$\frac{11-5}{5-2} = \frac{6}{3} = 2$

2. The table below gives the population of the rare coral mathematafish population as a function of months. The Environmental Protection Agency (EPA) is concerned the population is being threatened by an invasive species known as the fluted dropout shark. Through an intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathematafish population.

Months	0	1	2	3	4	5	6	7	8	9	10	11	12
Population	480	472	417	318	240	152	103	84	47	32	24	29	46

a. Calculate the average rate of change of the mathematafish population over the specific intervals.

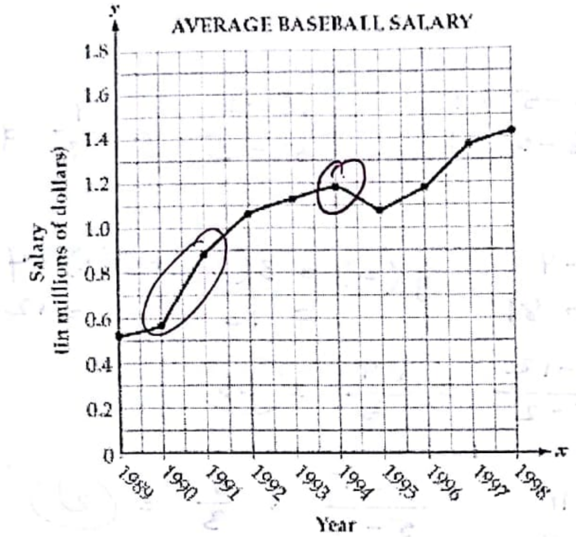
Interval	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Rate	−8	−55	−99	−78	−88	−49	−19	−37	−15	−8	+5	17

FISH/MONTH

b. When was the population decreasing fastest? 2ND TO 3RD MONTH.

c. During what month did you notice the largest effects of the EPA intervention? DISCUSS.

3. The graph below shows the average baseball salaries for the period 1989 to 1998.



a. What was the average salary in 1994? \$1.2 MILLION

b. What was the average rate of change in salaries from 1989 to 1998?

$$\frac{1.4 - 0.5}{1998 - 1989} = \frac{0.9}{9} = 0.1 \text{ MILLION } \$/\text{YR}$$

c. During what period of time did the average salaries *decrease*?

1994 – 1995

d. During what period of time did the average salaries *increase fastest*?

1990 – 1991