SECTION 6.1 - MIDPOINT + DISTANCE

* THE MIDPOINT OF A LINE SEGMENT IS A POINT THAT DIVIDES THE SEGMENT INTO EQUAL LENGTHS. AVERAGE X'S, AVERAGE Y'S

EXAMPLE. FIND THE MIDPOINT OF THE LINE SEGMENT WITH ENDPOINTS (4, -7) AND (-2, 3).

$x = \frac{4+(-2)}{2} = \frac{2}{2} = 1 \qquad y = \frac{-7+3}{2} = \frac{-4}{2} = -2$ $\boxed{M(1,-2)}$

EXAMPLE. THE DIAMETER OF A CIRCLE HAS ENDPOINTS (5, -11) AND (-7, 6). WHAT ARE THE COORDINATES OF THE CENTER OF THE CIRCLE?

$\frac{5+(-7)}{2} = \frac{-2}{2} = 1 \qquad \frac{-11+6}{2} = \frac{-5}{2} = -2.5$ $\boxed{C(1,-2.5)}$

APPLICATION. FIND THE MIDPOINT OF EACH LINE SEGMENT WITH THE GIVEN ENDPOINTS.

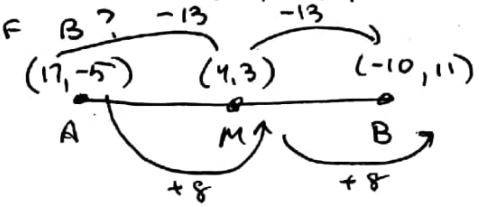
(A) (12, 7) AND (-2, 11)

$\frac{12+(-2)}{2} = \frac{10}{2} = 5 \qquad \frac{7+11}{2} = \frac{18}{2} = 9$ $\boxed{M(5,9)}$

(B) (-8, -3) AND (10, 9)

$\frac{-8+10}{2} = \frac{2}{2} = 1 \qquad \frac{-3+9}{2} = \frac{6}{2} = 3$ $\boxed{M(1,3)}$

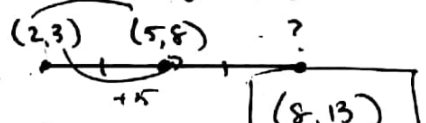
EXAMPLE. THE MIDPOINT OF $\overline{AB}$ IS M(4, 3). IF THE COORDINATES OF A ARE (17, -5), WHAT ARE THE COORDINATES OF B?



$\boxed{B(-10,11)}$

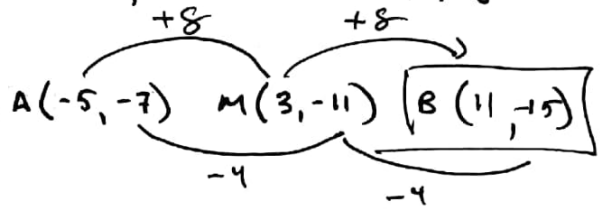
APPLICATION. FOR EACH OF THE FOLLOWING, FIND THE MISSING ENDPOINT.

(A)



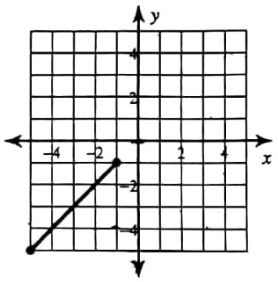
$5+3=8 \qquad 8+5=13$ $\boxed{(8,13)}$

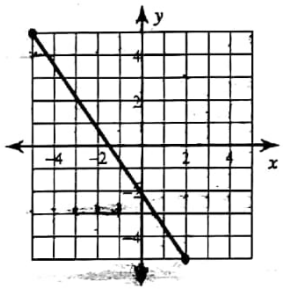
(B)

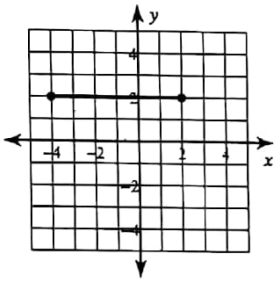


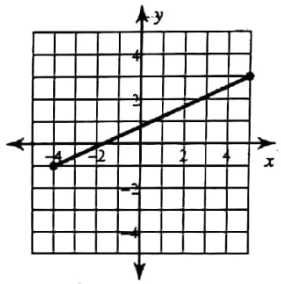
PRACTICE.

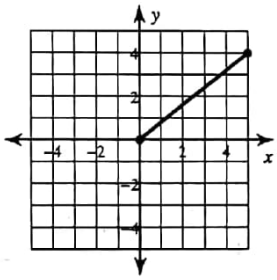
Find the midpoint of each line segment.

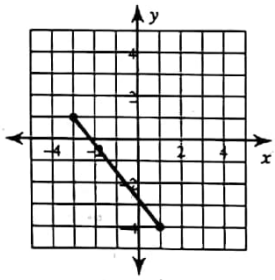
1)  (-3, -3)

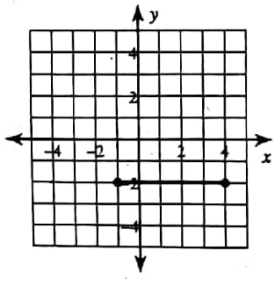
2)  (-1.5, 0)

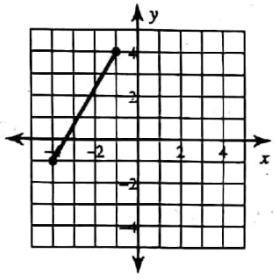
3)  (-1, 2)

4)  $(\frac{1}{2}, 1)$

5)  (2.5, 2)

6)  (-1, -1.5)

7)  (1.5, -2)

8)  (-2.5, 1.5)

2)

Find the midpoint of the line segment with the given endpoints.

9) (–4, 4), (5, –1)

$(\frac{1}{2}, 1\frac{1}{2})$

10) (–1, –6), (–6, 5)

$(-3.5, -\frac{1}{2})$

11) (2, 4), (1, –3)

$(1\frac{1}{2}, \frac{1}{2})$

12) (–4, 4), (–2, 2)

$(-3, 3)$

13) (5, 2), (–4, –3)

$(\frac{1}{2}, -\frac{1}{2})$

14) (–1, 1), (5, –5)

$(2, -2)$

15) (2, –1), (–6, 0)

$(-2, -\frac{1}{2})$

16) (–3.1, –2.8), (–4.92, –3.3)

$(-4.01, -3.05)$

17) (–5.1, –2), (1.4, 1.7)

$(-1.85, -0.15)$

18) (4.9, –1.3), (–5.2, –0.6)

$(-0.15, -0.95)$

19) (5.1, 5.71), (6, 3.6)

$(5.55, 4.655)$

20) (3.1, –2.1), (–0.52, –0.6)

$(1.29, -1.35)$

Find the other endpoint of the line segment with the given endpoint and midpoint.

21) Endpoint: (–1, 9), midpoint: (–9, –10)

$(-17, -29)$

22) Endpoint: (2, 5), midpoint: (5, 1)

$(8, -3)$

23) Endpoint: (5, 2), midpoint: (–10, –2)

$(-25, -6)$

24) Endpoint: (9, –10), midpoint: (4, 8)

$(-1, 26)$

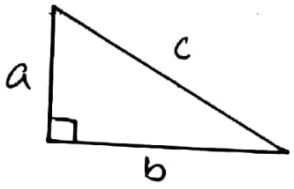
25) Endpoint: (–9, 7), midpoint: (10, –3)

$(29, -13)$

26) Endpoint: (–6, 4), midpoint: (4, 8)

$(14, 12)$

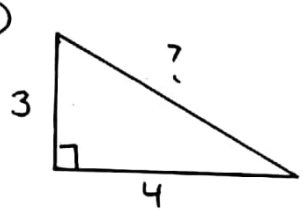
REVIEW OF THE PYTHAGOREAN THEOREM



$$a^2 + b^2 = c^2$$

EXAMPLES. FIND THE MISSING SIDE.

(A)



$3^2 + 4^2 = c^2$

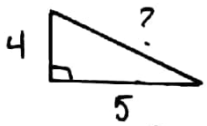
$9 + 16 = c^2$

$25 = c^2$

$c = \sqrt{25}$

$c = 5$

(B)



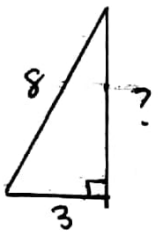
$4^2 + 5^2 = c^2$

$16 + 25 = c^2$

$c^2 = 41$

$c = \sqrt{41}$

(C)



$3^2 + b^2 = 8^2$

$9 + b^2 = 64$

$-9 \quad -9$

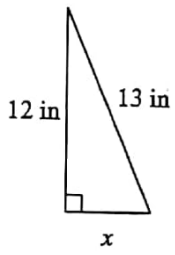
$b^2 = 55$

$b = \sqrt{55}$

APPLICATIONS.

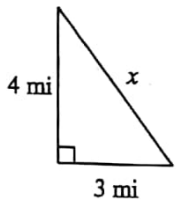
Find the missing side of each triangle. Round your answers to the nearest tenth if necessary.

1)



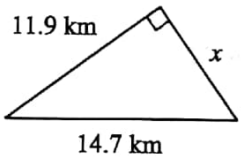
5

2)



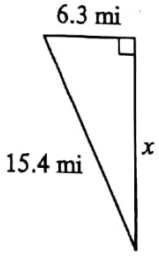
5

3)



8.6

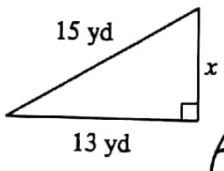
4)



14.1

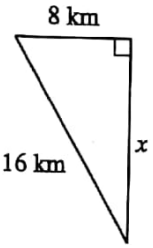
Find the missing side of each triangle. Leave your answers in simplest radical form.

5)



$2\sqrt{14}$

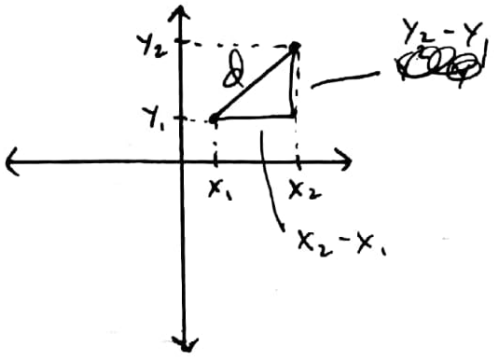
6)

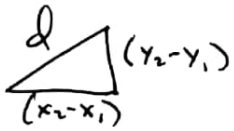


$8\sqrt{3}$

THE DISTANCE FORMULA

THE DISTANCE BETWEEN ANY 2 POINTS (x_1, y_1) AND (x_2, y_2) CAN BE FOUND USING THE DISTANCE FORMULA. THE DISTANCE FORMULA IS DERIVED FROM THE PYTHAGOREAN FORMULA.





$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

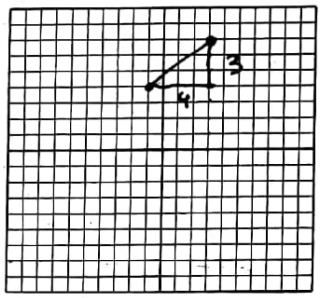
$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

EXAMPLES. FIND THE DISTANCE BETWEEN EACH PAIR OF POINTS.

$\sqrt{68}$: $68 = 2 \cdot 34$, $34 = 2 \cdot 17$

(A) $(3, 7)$ AND $(-1, 4)$

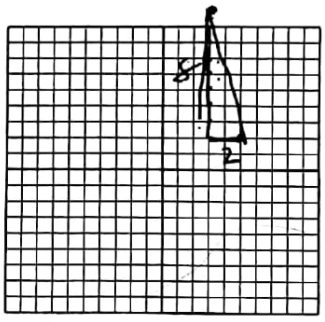
$$d = \sqrt{(3 - (-1))^2 + (7 - 4)^2}$$
$$= \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$



$3^2 + 4^2 = d^2$
$d^2 = 25$
$d = \sqrt{25} = 5$

(B) $(5, 2)$ AND $(3, 10)$

$$d = \sqrt{(5 - 3)^2 + (2 - 10)^2}$$
$$= \sqrt{2^2 + (-8)^2} = \sqrt{4 + 64} = \sqrt{68}$$
$$= 2\sqrt{17}$$



$8^2 + 2^2 = d^2$
$64 + 4 = d^2$
$d = \sqrt{68}$
$= 2\sqrt{17}$

APPLICATION. FIND THE DISTANCE BETWEEN EACH PAIR OF POINTS.

(1) $(5, 2)$ AND $(-2, -5)$

$$d = \sqrt{(5 - (-2))^2 + (2 - (-5))^2}$$
$$= \sqrt{(5 + 2)^2 + (2 + 5)^2}$$
$$= \sqrt{49 + 49} = \sqrt{2 \cdot 49}$$
$$= 7\sqrt{2}$$

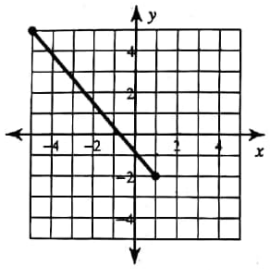
(2) $(5, 2)$ AND $(-2, 5)$

$$d = \sqrt{(5 - (-2))^2 + (2 - 5)^2}$$
$$= \sqrt{7^2 + (-3)^2}$$
$$= \sqrt{49 + 9} = \sqrt{58}$$

PRACTICE.

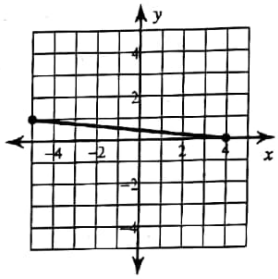
Find the distance between each pair of points.

1)



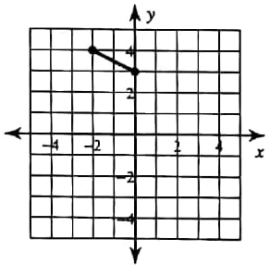
9.2

2)



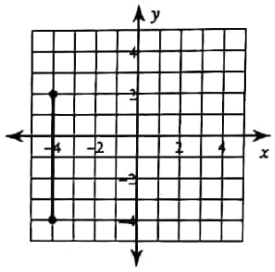
9.1

3)



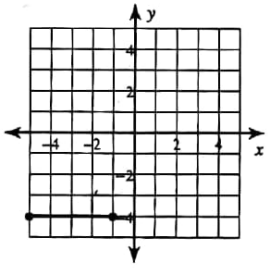
2.2

4)



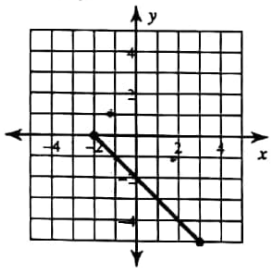
6

5)



4

6)



7.1

7) (–2, 3), (–7, –7) 11.2

8) (2, –9), (–1, 4) 13.3

9) (5, 9), (–7, –7) 20

10) (8, 5), (–1, 3) 9.2

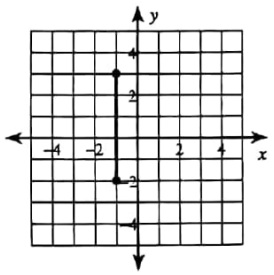
11) (–10, –7), (–8, 1) 8.2

12) (–6, –10), (–2, –10) 4

5

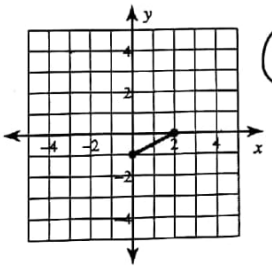
Find the distance between each pair of points.

13)



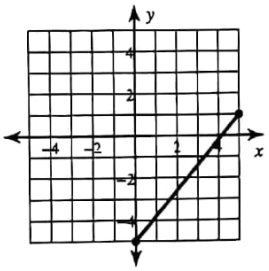
$\sqrt{5}$

14)



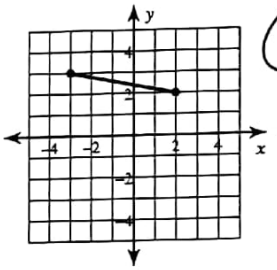
$\sqrt{5}$

15)



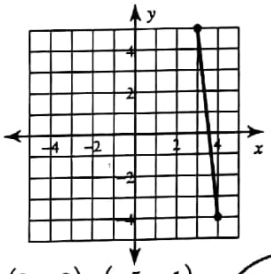
$\sqrt{61}$

16)



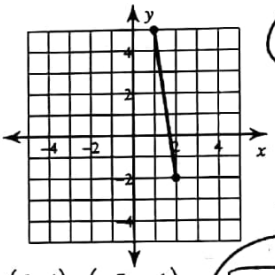
$\sqrt{26}$

17)



$\sqrt{82}$

18)



$5\sqrt{2}$

19) (0, –2), (–5, –1) $\sqrt{26}$

20) (6, 4), (–5, –1) $\sqrt{146}$

21) (3, 8), (9, 10) $2\sqrt{10}$

22) (10, 1), (9, –4) $\sqrt{26}$

23) (–8, 10), (–6, 7) $\sqrt{13}$

24) (–5, 6), (8, –4) $\sqrt{269}$