SECTION 4.6 – GRAPHING QUADRATICS

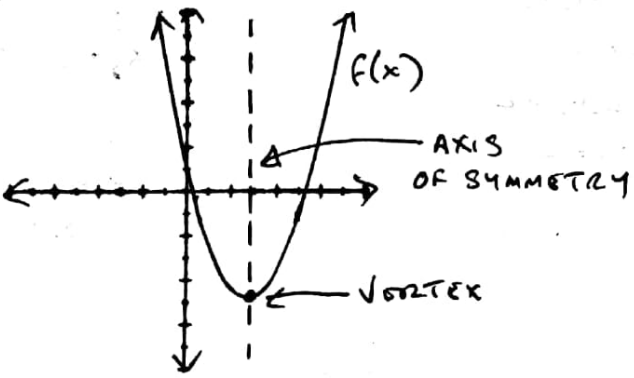
NAME: MAYO

★ QUADRATIC FUNCTIONS ARE NONLINEAR AND CAN BE WRITTEN IN THE FORM:

$$f(x) = ax^2 + bx + c$$

THIS FORM IS REFERRED TO AS STANDARD FORM.

THE SHAPE OF THE GRAPH OF A QUADRATIC FUNCTION IS CALLED A PARABOLA. PARABOLAS ARE SYMMETRIC ABOUT A CENTRAL LINE CALLED THE AXIS OF SYMMETRY. THE AXIS OF SYMMETRY INTERSECTS THE GRAPH AT ONLY ONE POINT, CALLED THE VERTEX.



PARENT FUNCTION: $f(x) = x^2$

STANDARD FORM: $f(x) = ax^2 + bx + c$

TYPE OF GRAPH: PARABOLA

AXIS OF SYMMETRY: $x = \frac{-b}{2a}$

Y-INTERCEPT: c

★ OPEN UP OR OPEN DOWN?

WHEN $a > 0$, THE PARABOLA OPENS UP ∪

AND THE VERTEX IS A MINIMUM.

WHEN $a < 0$, THE PARABOLA OPENS DOWN ∩

AND THE VERTEX IS A MAXIMUM.

EXAMPLE 1.

(A) $f(x) = -2x^2 + x - 1$

OPENS ↓; MAX

(B) $g(x) = 3x^2 - 5x + 2$

OPENS UP, MIN.

★ GRAPHING QUADRATIC FUNCTIONS

$f(x) = x^2 - 2x + 2$

(1) DECIDE IF PARABOLA OPENS UP OR DOWN.

$a = 1$ $\boxed{\text{OPENS UP}}$

(2) FIND Y-INTERCEPT.

$c = 2$ $\boxed{\text{Y-INT} = 2}$

(3) FIND THE EQUATION OF THE AXIS OF SYMMETRY.

$-\frac{b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

$\boxed{x = 1}$

(4) FIND THE VERTEX.

$f(1) = 1^2 - 2(1) + 2 = 1 - 2 + 2 = 1$

$V(1,1)$

(5) MAKE A T-CHART TO HELP GRAPH.

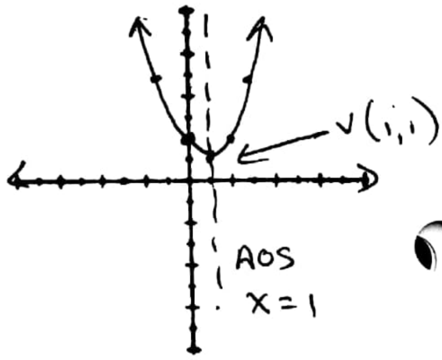
X	f(x)
-1	5
0	2
1	1
2	2
3	5

$(-1)^2 - (2)(-1) + 2$

$1 + 2 + 2$

$2^2 - 2(2) + 2$

$4 - 4 + 2$



APPLICATION 1

(1) GIVEN $f(x) = x^2 - 4x + 2$,

(A) UP OR DOWN? UP $a = 1$

(B) Y-INTERCEPT? 2

(C) EQUATION OF AOS? $-\frac{b}{2a} = \frac{-(-4)}{2(1)} = 2$

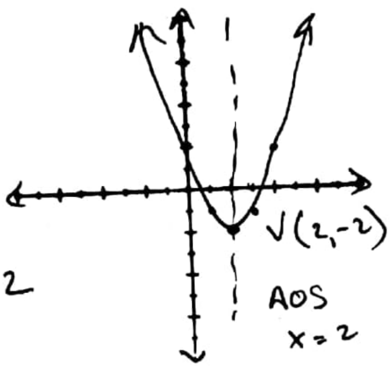
$x = 2$

(D) VERTEX? $2^2 - 4(2) + 2 = 4 - 8 + 2 = -2$ $(2,-2)$

(E) T-CHART.

x	f(x)
0	2
1	-1
2	-2
3	-1
4	2

(F) GRAPH



(2)

② GIVEN $g(x) = x^2 + 6x + 8$

Ⓐ UP OR DOWN? UP $a = 1$

Ⓑ Y-INTERCEPT? $c = 8$ $(0, 8)$

Ⓒ EQUATION OF AOS? $-\frac{b}{2a} = \frac{-6}{2(1)} = -3$

$x = -3$

Ⓓ VERTEX? $(-3)^2 + 6(-3) + 8 = 9 - 18 + 8 = -1$

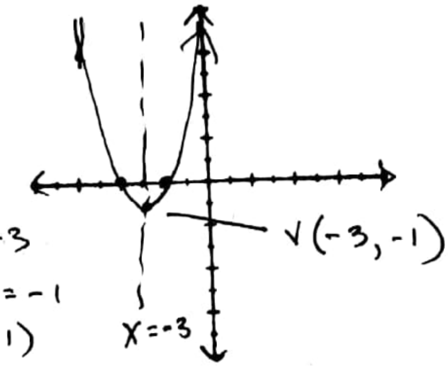
$V(-3, -1)$

Ⓔ T-CHART.

x	$g(x)$
-2	0
-3	-1
-4	0

$4 - 12 + 8 = 0$

Ⓕ GRAPH:



EXAMPLE 2. THE CHEERLEADERS OF N. MIDDLESEX H.S. LAUNCH T-SHIRTS INTO THE CROWD EVERY TIME THE TEAM SCORES A TOUCHDOWN. THE HEIGHT OF A T-SHIRT CAN BE MODELED BY THE FUNCTION $h(x) = -16x^2 + 48x + 6$, WHERE $h(x)$ REPRESENTS THE HEIGHT IN FEET OF THE T-SHIRT AFTER X SECONDS.

Ⓐ GRAPH THE FUNCTION

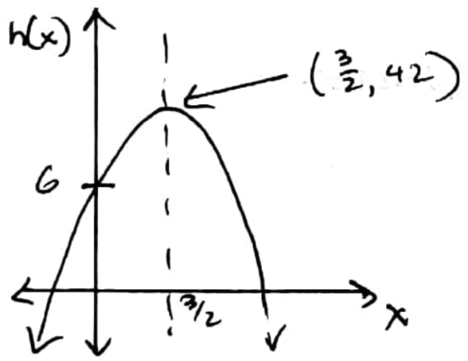
$a = -16 \Rightarrow$ OPENS ↓

$-\frac{b}{2a} = \frac{-48}{2(-16)} = \frac{-48}{-32} = +\frac{3}{2}$

$-16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 6 =$

$-16\left(\frac{9}{4}\right) + 48\left(\frac{3}{2}\right) + 6 =$

$-36 + 72 + 6 = 42$ $V\left(\frac{3}{2}, 42\right)$



Ⓑ AT WHAT HEIGHT WAS THE T-SHIRT LAUNCHED?

6 FT $\Rightarrow$ Y-INTERCEPT.

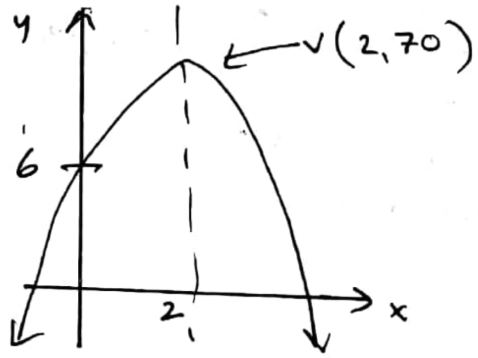
Ⓒ WHAT IS THE MAXIMUM HEIGHT OF THE T-SHIRT AND AT WHAT TIME WAS IT REACHED?

MAX HT = 42 ft @ $\frac{3}{2}$ SECONDS.

(VERTEX)

APPLICATION 2. EMELIO IS COMPETING IN THE JAVELIN THROW. THE HEIGHT OF THE JAVELIN CAN BE MODELED BY THE EQUATION $y = -16x^2 + 64x + 6$, WHERE Y REPRESENTS THE HEIGHT IN FEET OF THE JAVELIN AFTER X SECONDS.

(A) GRAPH THE PATH OF THE JAVELIN.



$a = -16$ OPENS ↓

$\frac{-b}{2a} = \frac{-64}{2(-16)} = \frac{-64}{-32} = 2$

$-16(2)^2 + 64(2) + 6 =$

$-64 + 128 + 6 = 70$

$V(2,70)$

(B) AT WHAT HEIGHT IS THE JAVELIN THROWN?

6 ft (Y-INTERCEPT)

(C) WHAT IS THE MAXIMUM HEIGHT OF THE JAVELIN AND AT WHAT TIME WAS IT REACHED?

70 FT, 2 SECONDS (VERTEX)

PRACTICE!

(1) FIND THE VERTEX, EQUATION OF THE AXIS OF SYMMETRY, AND THE Y-INTERCEPT OF THE GRAPH OF EACH FUNCTION.

(A) $y = -3x^2 + 6x - 1$

$\frac{-b}{2a} = \frac{-6}{2(-3)} = 1$

$-3(1)^2 + 6(1) - 1 = -3 + 6 - 1 = 2$

V(1,2)
AOS: x = 1
Y-INT = -1

(B) $y = -x^2 + 2x + 1$

$\frac{-b}{2a} = \frac{-2}{2(-1)} = 1$

$-(1)^2 + 2(1) + 1 = -1 + 2 + 1 = 0$

$-0^2 + 2(0) + 1 = 1$

AOS: x = 1
Y-INT: 1
V(0,1)

(C) $y = x^2 - 4x + 5$

$\frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$

$2^2 - 4(2) + 5 =$

$4 - 8 + 5 = 1$

V(2,1)
AOS: x = 2
Y-INT = 5

4

(D) $y = 4x^2 - 8x + 9$

$\frac{-b}{2a} = \frac{-(-8)}{2(4)} = \frac{8}{8} = 1$

$4(1)^2 - 8(1) + 9 = 4 - 8 + 9 = 5$

V(1,5)
AOS: x = 1
Y-INT: 9

2) FOR EACH FUNCTION, DETERMINE IF IT HAS A MAXIMUM OR A MINIMUM, STATE THE MAXIMUM OR MINIMUM VALUE, STATE THE DOMAIN + RANGE.

Ⓐ $y = -x^2 + 4x - 3$ MAX

$\frac{-b}{2a} = \frac{-4}{2(-1)} = 2$ $-(2)^2 + 4(2) - 3)$
$-4 + 8 - 3 = 1$

MAX = 1

D: ℝ R: $y \leq 1$

Ⓑ $y = -x^2 - 2x + 2$ MAX

$\frac{-b}{2a} = \frac{-(-2)}{2(-1)} = \frac{2}{-2} = -1$

$-(-1)^2 - 2(-1) + 2 = -1 + 2 + 2 = 3$ MAX = 3

D: ℝ R: $y \leq 3$

Ⓒ $y = 3x^2 + 6x + 3$ MIN

$\frac{-b}{2a} = \frac{-6}{2(3)} = -1$ MIN = 0

$3(-1)^2 + 6(-1) + 3 =$
$3 - 6 + 3 = 0$

D: ℝ

R: $y \geq 0$

Ⓓ $y = 2x^2 + 8x - 6$ MIN

$\frac{-b}{2a} = \frac{-8}{2(2)} = -2$ MIN = −14

$2(-2)^2 + 8(-2) - 6 =$
$8 - 16 - 6 = -14$

D: ℝ

R: $y \geq -14$

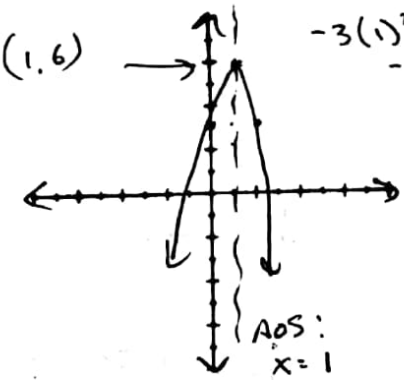
③ GRAPH EACH FUNCTION. CLEARLY LABEL: Ⓐ AOS, Ⓑ VERTEX.

Ⓐ $f(x) = -3x^2 + 6x + 3$

$\frac{-b}{2a} = \frac{-6}{2(-3)} = 1$

$-3(1)^2 + 6(1) + 3$
$-3 + 6 + 3 = 6$
$V(1, 6)$
AOS: $x = 1$

V(1, 6)

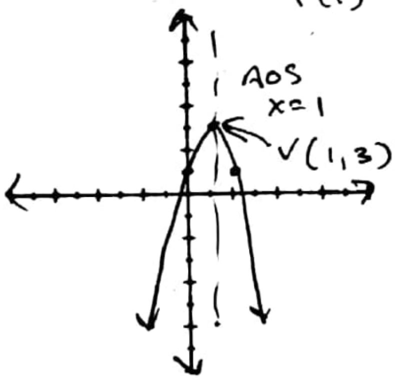


x	f(x)
−1	−6
0	3
1	6
2	3
3	−6

Ⓑ $f(x) = -2x^2 + 4x + 1$

$\frac{-b}{2a} = \frac{-4}{2(-2)} = 1$

$f(1) = -2 + 4 + 1$
$= 3$
$V(1, 3)$

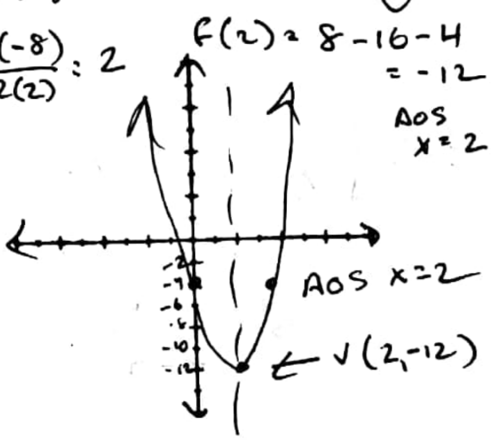


Ⓒ $f(x) = 2x^2 - 8x - 4$

$\frac{-b}{2a} = \frac{-(-8)}{2(2)} = 2$

$f(2) = 8 - 16 - 4$
$= -12$

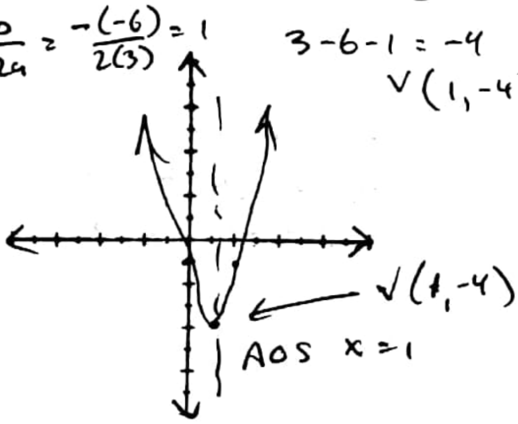
AOS
$x = 2$



Ⓓ $f(x) = 3x^2 - 6x - 1$

$\frac{-b}{2a} = \frac{-(-6)}{2(3)} = 1$

$3 - 6 - 1 = -4$
$V(1, -4)$



(4) A FOOTBALL IS KICKED UP FROM THE GROUND LEVEL AT AN INITIAL UPWARD VELOCITY OF 90 FEET PER SECOND. THE EQUATION $h = -16t^2 + 90t$ GIVES THE HEIGHT AFTER t SECONDS.

(A) WHAT IS THE HEIGHT OF THE BALL AFTER 1 SECOND?

$h(1) = -16(1)^2 + 90(1) = -16 + 90 = \boxed{74 \text{ ft}}$

(B) WHEN IS THE BALL 126 FEET HIGH?

$126 = -16t^2 + 90t \qquad -16t^2 + 90t - 126 = 0$ CALC.

3 SECS

(C) WHEN IS THE HEIGHT OF THE BALL 0 FEET?

$-16t^2 + 90t = 0$

$-2t(8t - 45) = 0$

$-2t = 0$

$t = 0$

$8t - 45 = 0$

$8t = 45$

$t = \frac{45}{8}$ sec

(D) HOW HIGH DOES THE BALL GO?

$-\frac{b}{2a} = \frac{-90}{2(-16)} = \frac{-90}{-32} = \frac{45}{16}$ | $-16\left(\frac{45}{16}\right)^2 + 90\left(\frac{45}{16}\right) = 126\frac{9}{16}$ ft

(5) LET $f(x) = x^2 - 9$.

(A) WHAT IS THE DOMAIN OF $f(x)$? D: $\mathbb{R}$

(B) WHAT IS THE RANGE OF $f(x)$? R: $x \geq -9$

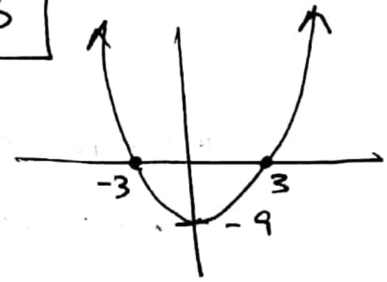
(C) FOR WHAT VALUES IS $f(x)$ NEGATIVE?

$x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$x = 3 \quad x = -3$

$\boxed{-3 < x < 3}$



(HINT: GRAPH $f(x)$)

(6)