

SECTION 4.5 (CONTINUED) - DIFFERENCES OF SQUARES

REVIEW: FIND THE SUMS OR PRODUCTS!

$$(1) (3x^2 + 2x + 1) + (x^2 - 3x - 6) = 4x^2 - x - 5$$

$$(2) (3x^2 + 1)(2x + 2) = 6x^3 + 6x^2 + 2x + 2$$

$$(3) (x + 1)(x - 1) = x^2 - 1$$

$$(4) (x + 2)(x - 2) = x^2 - 4$$

DIFFERENCE OF SQUARES

THE ANSWERS TO QUESTIONS (3) AND (4) ABOVE ARE REFERRED TO AS A DIFFERENCE OF SQUARES.

- IN NUMBER (3), WHAT WAS MULTIPLIED TOGETHER TO GET $x^2 - 1$? $(x + 1)(x - 1)$

- IN NUMBER (4), WHAT WAS MULTIPLIED TOGETHER TO GET $x^2 - 4$? $(x + 2)(x - 2)$

FACTORIZING A DIFFERENCE OF SQUARES!

$$a^2 - b^2 = (a + b)(a - b) \text{ or } (a - b)(a + b)$$

EXAMPLE 1. FACTOR EACH POLYNOMIAL.

$$(A) 16h^2 - 9a^2 = (4h + 3a)(4h - 3a)$$

$$(B) 121 - 4b^2 = (11 + 2b)(11 - 2b)$$

$$(C) 27g^3 - 3g = 3g(9g^2 - 1) = 3g(3g - 1)(3g + 1)$$

EXERCISES 1.10 - 2. FACTOR

$$(1) 81 - c^2 = (9 - c)(9 + c)$$

$$(2) 64x^2 - y^2 = (8x - y)(8x + y)$$

$$(3) 9x^3 - 4x = x(9x^2 - 4) = x(3x - 2)(3x + 2)$$

$$(4) -4y^3 + 9y = -y(4y^2 - 9) = -y(2y - 3)(2y + 3)$$

$$9y^2 - 4y^3 = y(9 - 4y^2) = y(3 - 2y)(3 + 2y)$$

EXAMPLE 2. FACTOR EACH POLYNOMIAL:

$$(A) b^4 - 16 = (b^2 - 4)(b^2 + 4) = (b - 2)(b + 2)(b^2 + 4)$$

$$(B) 625 - x^4 = (25 - x^2)(25 + x^2) = (5 - x)(5 + x)(25 + x^2)$$

APPLICATION 2. FACTOR:

$$(1) y^4 - 1 = (y^2 - 1)(y^2 + 1) = (y - 1)(y + 1)(y^2 + 1)$$

$$(2) 4a^4 - b^4 = (2a^2 - b^2)(2a^2 + b^2)$$

$$(3) 81 - x^4 = (9 - x^2)(9 + x^2) = (3 - x)(3 + x)(9 + x^2)$$

$$(4) 16y^4 - 1 = (4y^2 - 1)(4y^2 + 1) = (2y + 1)(2y - 1)(4y^2 + 1)$$

$$(5) x^2 + 2x - 8 = (x + 4)(x - 2)$$

(2)

EXAMPLE 3. FACTOR EACH POLYNOMIAL

$$\textcircled{A} \quad 5x^5 - 45x = 5x(x^4 - 9) \\ = 5x(x^2 - 3)(x^2 + 3)$$

$$\textcircled{B} \quad (7x^3 + 21x^2) - (7x + 21) \quad \text{GCF: } 7$$

$$\cancel{7x^2(x+3)} - \cancel{7(x+3)} \quad 7(x^3 + 3x^2 - x - 3)$$

$$7((x^3 + 3x^2) - (x + 3)) =$$

$$7(x^2(x+3) - 1(x+3)) = 7(x^2 - 1)(x+3) = \boxed{7(x-1)(x+1)(x+3)}$$

APPLICATION 3. FACTOR:

$$\textcircled{1} \quad 2y^4 - 50 = 2(y^4 - 25) = 2(y^2 - 5)(y^2 + 5)$$

$$\textcircled{2} \quad 6x^4 - 96 = 6(x^4 - 16) = 6(x^2 - 4)(x^2 + 4) \\ = 6(x-2)(x+2)(x^2 + 4)$$

$$\textcircled{3} \quad 2m^3 + m^2 - 50m - 25 \\ (2m^3 + m^2) - (50m + 25) = \\ m^2(2m+1) - 25(2m+1) = (m^2 - 25)(2m+1) \\ = (m-5)(m+5)(2m+1)$$

$$\textcircled{4} \quad (r^3 + 6r^2) + (11r + 66) \\ r^2(r+6) + 11(r+6) \\ (r^2 + 11)(r+6)$$

EXERCISES

① FACTOR:

(A) $x^2 - 121 = (x-11)(x+11)$

(B) $r^4 - k^4 = (r^2 - k^2)(r^2 + k^2) = (r-k)(r+k)(r^2 + k^2)$

(C) $6n^2 - 6 = 6(n^2 - 1) = 6(n-1)(n+1)$

(D) $h^3 - 100h = h(h^2 - 100) = h(h-10)(h+10)$

(E) $3r^3 - 192r = 3r(r^2 - 64) = 3r(r-8)(r+8)$

(F) $3x^3 - 7x^2 - 3x + 7 = (3x^3 - 7x^2) - (3x - 7) =$

(G) $a^2 - 49 = (a-7)(a+7)$ $x^2(3x-7) - 1(3x-7) = (x^2-1)(3x-7)$
 $= (x-1)(x+1)(3x-7)$

(H) $(r^3 - 5r^2) - (100r + 500) = r^2(r-5) - 100(r-5)$
 $(r^2 - 100)(r-5) = \boxed{(r-10)(r+10)(r-5)}$

② SOLVE:

(A) $x^2 - 4 = 0 \quad (x-2)(x+2) = 0 \quad \{-2, 2\}$

(B) $2x^2 - 32 = 0 \quad x^2 - 16 = 0 \quad (x-4)(x+4) = 0 \quad \{-4, 4\}$

(C) $x^2 + 2x - 8 = 0 \quad (x+4)(x-2) = 0 \quad \{-4, 2\}$

(D) $x^2 + 6x = 27 \quad x^2 + 6x - 27 = 0$
 $(x+9)(x-3) = 0 \quad \{-9, 3\}$

(E) $5x^2 - 22x + 8 = 0 \quad ac = 40 \quad -2, -20$
 $5x^2 - 20x - 2x + 8 = 0 \quad (5x^2 - 20x) - (2x - 8) = 0$

(F) $4x^2 - 16x = -15$
 $4x^2 - 16x + 15 = 0 \quad ac = 60$