

## SECTION 4.4 - FACTORING BY GREATEST COMMON FACTOR (GCF)

EXAMPLE 1. FACTOR  $27x^2 + 18x$

STEP 1: FIND THE GREATEST COMMON FACTOR OF EACH TERM.  
 $3 \cdot 3 \cdot 3 \cdot x \cdot x$      $2 \cdot 3 \cdot 3 \cdot x$     GCF =  $9x$

STEP 2: WRITE EACH TERM AS THE PRODUCT OF THE GCF AND THE REMAINING FACTORS. FACTOR OUT THE GCF.  
 $9x(3x + 2)$

EXAMPLE 2. FACTOR  $-4a^2b - 8ab^2 + 2ab$   
 $2ab(-2a - 4b + 1)$

APPLICATION 1. FACTOR:

①  $15w - 3v$   
 $3(5w - v)$

②  $7u^2 + 21ut - ut$   
 $ut(7ut + 21 - 1) =$   
 $ut(7ut + 20)$

PRACTICE: FACTOR:

①  $21b - 15a$

$3(7b - 5a)$

②  $14c^2 + 2c$

$2c(7c + 1)$

③  $10g^2h^2 + 9gh^2 - g^2h$

$gh(10gh + 9h - g)$

④  $12jk^2 + 6j^2k + 2j^2k^2$

$2jk(6k + 3j + jk)$

⑤  $21x^2 - 7x + 14$

$7(3x^2 - x + 2)$

⑥  $2x^2 + 4x$

$2x(x + 2)$

EXAMPLE 3. ANOTHER TYPE OF FACTORING IS CALLED FACTOR BY GROUPING. IT IS USED WHEN A POLYNOMIAL HAS 4 OR MORE TERMS AND TERMS HAVE COMMON FACTORS THAT CAN BE GROUPED TOGETHER.

$$\textcircled{A} \text{ FACTOR: } (4qr + 8r) + (3q + 6)$$

$$4r(q + 2) + 3(q + 2)$$

$$(4r + 3)(q + 2)$$

$$\textcircled{B} \text{ FACTOR: } (fg + 5g) + (4f + 20)$$

$$g(f + 5) + 4(f + 5)$$

$$(g + 4)(f + 5)$$

~~(f)~~

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APPLICATION 2. FACTOR:

$$\textcircled{1} (6x^2 - 4x) + (3x + 2)$$

$$2x(3x - 2) + 1(3x + 2)$$

$$\textcircled{2} (9x^2 - 3xy) + (6x - 2y)$$

$$3x(3x - y) + 2(3x - y)$$

$$(3x + 2)(3x - y)$$

$$\textcircled{3} (6xy - 8x) + (5y - 20)$$

$$2x(3y - 4) + 5(3y - 4)$$

$$(2x + 5)(3y - 4)$$

$$\textcircled{4} (-6mp + 4m) + (18p - 12)$$

$$2m(-3p + 2) + 6(3p - 2)$$

$$2m(-3p + 2) - 6(-3p + 2)$$

$$(2m - 6)(-3p + 2)$$

EXAMPLE 4.  $\textcircled{A}$  FACTOR:  $(rn + 5n) - (r + 5)$

$$n(r + 5) - 1(r + 5)$$

$$(n - 1)(r + 5)$$

$$\textcircled{B} \text{ FACTOR: } (3np + 15p) - (4n + 20)$$

$$3p(n + 5) - 4(n + 5)$$

$$(3p - 4)(n + 5)$$

(2)

APPLICATION 3. FACTOR:

①  $(12a^2 - 15ab) - (16a + 20b)$

$3a(4a - 5b) - 4a(4a - 5b) = (3a - 4a)(4a - 5b)$

②  $(-6mp + 4m) - (8p + 12)$

$2m(-3p + 2) - 6(3p + 2)$

CANNOT FACTOR.

PRACTICE. FACTOR:

①  $64 - 4ab$

$4(16 - ab)$

②  $4d^2 + 16$

$4(d^2 + 4)$

③  $6r^2 + 3r + 2$

$3r + (2r + 2)$

④  $15ad + 30a^2d^2$

$15ad(1 + 2ad)$

⑤  $9a^3d^2 - 6ad^3$

$3ad^2(3a^2 - 2d)$

⑥  $8p^2r^2 - 24pr^3 + 16pr$

$8pr(pr - 3r^2 + 2)$

⑦  $(x^2 + 4x) + (2x + 8)$

$x(x + 4) + 2(x + 4)$

$(x + 2)(x + 4)$

⑧  $(2a^2 + 3a) + (6a + 9)$

$a(2a + 3) + 3(2a + 3)$

$(a + 3)(2a + 3)$

⑨  $(4x^2 - 3xy) + (6x - 2y)$

$3x(3x - y) + 2(3x - y)$

$(3x + 2)(3x - y)$

⑩  $(2a^2 - 4a) + (a - 2)$

$2a(a - 2) + 1(a - 2)$

$(2a + 1)(a - 2)$

## SOLVING EQUATIONS BY FACTORING

CONSIDER:  $3(0) = 0$      $-3(2(0)) = 0$      $0(0.25) = 0$

ZERO PRODUCT PROPERTY: IF THE PRODUCT OF 2 FACTORS EQUALS 0, THEN AT LEAST ONE OF THE FACTORS MUST BE 0.

EXAMPLE 5. SOLVE EACH EQUATION:

(A)  $(2d + 6)(3d - 15) = 0$

$$2d + 6 = 0$$

$$2d = -6$$

$$d = -3$$

$$3d - 15 = 0$$

$$3d = 15$$

$$d = 5$$

$$d = \{-3, 5\}$$

(B)  $c^2 = 3c$

$$c^2 - 3c = 0$$

$$c(c - 3) = 0$$

$$c = 0$$

$$c - 3 = 0$$

$$c = 3$$

$$c = \{0, 3\}$$

(C)  $8b^2 - 40b = 0$

$$8b(b - 5) = 0$$

$$8b = 0$$

$$b = 0$$

$$b - 5 = 0$$

$$b = 5$$

$$b = \{0, 5\}$$

APPLICATION 4. SOLVE + CHECK.

(1)  $x^2 - 10x = 0$

$$x(x - 10) = 0$$

$$x = 0 \quad x = 10$$

$$\{0, 10\}$$

(2)  $3x(x + 10) = 0$

$$3x = 0 \quad | \quad x + 10 = 0$$

$$x = 0 \quad | \quad x = -10$$

$$\{0, -10\}$$

(3)  $20x^2 - 15x = 0$

$$5x(4x - 3) = 0$$

$$5x = 0 \quad | \quad 4x - 3 = 0$$

$$x = 0$$

$$4x = 3$$

$$x = \frac{3}{4}$$

$$\left\{0, \frac{3}{4}\right\}$$

(4)  $r^2 = 14r$

$$r^2 - 14r = 0$$

$$r(r - 14) = 0$$

$$r = 0$$

$$r - 14 = 0$$

$$r = 14$$

$$\{0, 14\}$$

(A)

PRACTICE. SOLVE + CHECK.

①  $x(x-8) = 0$

$$x=0 \quad | \quad x-8=0$$

$$x=8 \quad \{0, 8\}$$

③  $(m-3)(m+5) = 0$

$$m-3=0 \quad | \quad m+5=0$$

$$m=3 \quad | \quad m=-5$$

$$\{3, -5\}$$

⑤  $x^2 - 5x = 0$

$$x(x-5) = 0 \quad \{0, 5\}$$

$$x=0 \quad | \quad x-5=0$$

$$x=5$$

⑦  $3a^2 = 6a$

$$3a^2 - 6a = 0 \quad \{0, 2\}$$

$$3a(a-2) = 0$$

$$3a=0 \quad | \quad a-2=0$$

$$a=0 \quad | \quad a=2$$

⑨  $4x(x+4) = 0 \quad \{0, -4\}$

$$4x=0 \quad | \quad x+4=0$$

$$x=0 \quad | \quad x=-4$$

⑪  $8x^2 = -26x$

$$8x^2 + 26x = 0 \quad \{0, -\frac{13}{4}\}$$

$$2x(4x+13) = 0$$

$$2x=0 \quad | \quad 4x+13=0$$

$$x=0 \quad | \quad x=-\frac{13}{4}$$

⑬  $(a+6)(3a-7) = 0$

$$a+6=0 \quad | \quad 3a-7=0$$

$$a=-6 \quad | \quad 3a=7$$

$$a=\frac{7}{3}$$

$$\{-6, \frac{7}{3}\}$$

②  $b(b+12) = 0$

$$b=0 \quad | \quad b+12=0$$

$$b=-12 \quad \{0, -12\}$$

④  $(a-9)(2a+1) = 0$

$$a-9=0 \quad | \quad 2a+1=0$$

$$a=9 \quad | \quad 2a=-1$$

$$a=-\frac{1}{2} \quad \{-\frac{1}{2}, 9\}$$

⑥  $y^2 + 3y = 0$

$$y(y+3) = 0 \quad \{0, -3\}$$

$$y=0 \quad | \quad y+3=0$$

$$y=-3$$

⑧  $2x^2 = 3x$

$$2x^2 - 3x = 0 \quad \{0, \frac{3}{2}\}$$

$$x(2x-3) = 0$$

$$x=0 \quad | \quad 2x-3=0$$

$$2x=3 \quad x=\frac{3}{2}$$

⑩  $9x^2 = 27x$

$$9x^2 - 27x = 0 \quad \{0, 3\}$$

$$9x(x-3) = 0$$

$$9x=0 \quad | \quad x-3=0$$

$$x=0 \quad | \quad x=3$$

⑫  $x(x-32) = 0$

⑭  $(4y+8)(3y-4) = 0$

$$4y+8=0 \quad | \quad 3y-4=0$$

$$4y=-8 \quad | \quad 3y=4$$

$$y=-2 \quad | \quad y=\frac{4}{3}$$

$$\{-2, \frac{4}{3}\}$$

⑮