

Section 4.1 – Adding and Subtracting Polynomials

A **polynomial** consists of constants and variables joined together by addition, subtraction, and/or multiplication. The constants and variables are grouped together in one or more terms, each of which can be an individual number, a single variable, or a product of numbers and/or variables with exponents that are non-negative integers.

Example 1.

a. $6a^2 + 14a - 9$

POLYNOMIAL ✓

b. $200(7)^x + 1$

NOT A POLYNOMIAL (EXPONENTIAL)

c. $7x^{-1}$

NOT A POLYNOMIAL (NEGATIVE EXP.)

Polynomials with 1, 2, or 3 terms can be grouped into categories.

A MONOMIAL has 1 term, for example: $3x^2$

A BINOMIAL has 2 terms, for example: $4x^3 + x$

A TRINOMIAL has 3 terms, for example: $3x^2 + 2x - 1$

The degree of a polynomial is the highest power of any variable in the polynomial.

Polynomials can also be categorized by their degree. The degree of a monomial with one variable is equal to the value of the variable's exponent.

2 VARS OR MORE ADD DEGREES + TAKE HIGHEST DEGREE OF ANY TERM.

Degree	Name	Example
0	CONSTANT	8
1	LINEAR	$3x + 1$
2	QUADRATIC	$2x^2 - 4x + 1$
3	CUBIC	$4x^3 - 2x$
4	QUARTIC	$3x^4 + 2x + 1$
5	QUINTIC	$4x^5 + 3x^2 + 2$
6 or more	6th DEGREE, 7th DEGREE, etc	_____

The **standard form** of a polynomial contains no like terms and its terms are written in descending order of degree. In other words, the exponents should go from greatest to least.

Example 2.

a. $4a - 2a^2 + 5a^3$

$5a^3 - 2a^2 + 4a$

b. $3x + 2x^2 - x - 7$

$2x^2 + 3x - x - 7$
 $2x^2 + 2x - 7$

Adding Polynomials – Adding polynomials involves combining like terms.

Example 3. Find the sum. Write the answer in standard form.

a. $(2x^2 + 5x - 7) + (3 - 4x^2 + 6x)$

$$2x^2 - 4x^2 + 5x + 6x - 7 + 3$$

$$\boxed{-2x^2 + 11x - 4}$$

b. $(3y + y^3 - 5) + (4y^2 - 4y + 2y^3 + 8)$

$$y^3 + 2y^3 + 4y^2 + 3y - 4y - 5 + 8$$

$$\boxed{3y^3 + 4y^2 - y + 3}$$

Application 1. Find the sum. Write the answer in standard form.

a. $(5x^2 - 3x + 4) + (6x - 3x^2 - 3)$

$$5x^2 - 3x^2 - 3x + 6x + 4 - 3 = \boxed{2x^2 + 3x + 1}$$

b. $(y^4 - 3y + 7) + (2y^3 + 2y - 2y^4 - 11)$

$$= y^4 - 2y^4 + 2y^3 - 3y + 2y + 7 - 11$$
$$= \boxed{-y^4 + 2y^3 - y - 4}$$

Subtracting Polynomials. You can subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, write the opposite of each term.

Example 4. Find each difference. Write the answer in standard form.

a. $(3 - 2x + 2x^2) - (4x - 5 + 3x^2)$

$$(3 - 2x + 2x^2) + (-4x + 5 - 3x^2) =$$

$$2x^2 - 3x^2 - 2x - 4x + 3 + 5 =$$

$$\boxed{-x^2 - 6x + 8}$$

b. $(7p + 4p^3 - 8) - (3p^2 + 2 - 9p)$

$$(7p + 4p^3 - 8) + (-3p^2 - 2 + 9p) =$$

$$4p^3 - 3p^2 + 7p + 9p - 8 - 2 = \boxed{4p^3 - 3p^2 + 16p - 10}$$

Application 2. Find each difference. Write the answer in standard form.

1. $(4x^3 - 3x^2 + 6x - 4) - (-2x^3 + x^2 - 2)$

$$(4x^3 - 3x^2 + 6x - 4) + (2x^3 - x^2 + 2) =$$

$$4x^3 + 2x^3 - 3x^2 - x^2 + 6x - 4 + 2 = \boxed{6x^3 - 4x^2 + 6x - 2}$$

2. $(8y - 10 + 5y^2) - (7 - y^3 + 12y)$

$$(8y - 10 + 5y^2) + (-7 + y^3 - 12y) =$$

$$y^3 + 5y^2 + 8y - 12y - 10 - 7 = \boxed{y^3 + 5y^2 - 4y - 17}$$

Example 5. Simplify. Write final answer in standard form.

a. $(4x + 2y - 6z) + (5y - 2z + 7x) + (-9z - 2x - 3y)$

$$\cancel{4x} + \cancel{2y} - 6z + \cancel{5y} - 2z + \cancel{7x} - 9z - \cancel{2x} - \cancel{3y}$$

$$\boxed{9x + 4y - 17z}$$

b. $(3n^2 + 3n - 10) - (4n^2 - 5n) + (4n^3 - 3n^2 - 9n + 4)$

$$\cancel{3n^2} + \cancel{3n} - 10 - \cancel{4n^2} + \cancel{5n} + \cancel{4n^3} - \cancel{3n^2} - \cancel{9n} + 4 =$$

$$\boxed{4n^3 - 4n^2 - n - 6}$$

Practice.

1. $(2c^2 + 6c + 4) + (5c^2 - 7) = \boxed{7c^2 + 6c - 3}$

2. $(2x + 3x^2) - (7 - 8x^2) = 3x^2 + 8x^2 + 2x - 7 = \boxed{11x^2 + 2x - 7}$

3. $(4a - 5b^2 + 3) + (6 - 2a + 3b^2) = 4a - 2a - 5b^2 + 3b^2 + 3 + 6 = \boxed{2a - 2b^2 + 9}$

4. $(4rx - 8r^2x + x^2) - (6rx^2 + 5rx - 2x^2)$
 $4rx - 5rx + -8r^2x - 6rx^2 + x^2 + 2x^2$
 $\boxed{-rx - 8r^2x - 6rx^2 + 3x^2}$

5. Classify each polynomial according to its degree and number of terms.

a. $4x - 3x^2 + 5$ TRINOMIAL QUADRATIC

b. $11z^3$ MONOMIAL CUBIC

c. $10x - 4x^2 + 6x^3$ TRINOMIAL CUBIC

Skills Practice

Adding and Subtracting Polynomials

Find each sum or difference.

$$1. (2x + 3y) + (4x + 9y) = 6x + 12y$$

$$2. (6s + 5t) + (4t + 8s) = 14s + 9t$$

$$3. (5a + 9b) - (2a + 4b) = 3a + 5b$$

$$4. (11m - 7n) - (2m + 6n) = 9m - 13n$$

$$5. (m^2 - m) + (2m + m^2) = 2m^2 + m$$

$$6. (x^2 - 3x) - (2x^2 + 5x) = -x^2 - 8x$$

$$7. (d^2 - d + 5) - (2d + 5) = d^2 - 3d$$

$$8. (2h^2 - 5h) + (7h - 3h^2) = -h^2 + 2h$$

$$9. (5f + g - 2) + (-2f + 3)$$

$$3f + g + 1$$

$$10. (6k^2 + 2k + 9) + (4k^2 - 5k)$$

$$10k^2 - 3k + 9$$

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial*, *binomial*, or *trinomial*.

$$11. 5mt + t^2 \quad \text{BIN.} \quad \underline{2}$$

$$12. 4by + 2b - by \quad \text{TRI.} \quad \underline{2}$$

$$13. -32 \quad \text{MON} \quad \underline{0}$$

$$14. \frac{3x}{7} = \text{NOT A POLYNOMIAL.}$$

$$\frac{3}{7}x \quad \text{MONOMIAL} \quad \underline{1}$$

$$15. 5x^2 - 3x^{-4} \quad \text{NOT A POLY.}$$

$$16. 2c^2 + 8c + 9 - 3 \quad \text{TRI} \quad \underline{2}$$

Write each polynomial in standard form. Identify the leading coefficient.

$$17. 3x + 1 + 2x^2$$

$$2x^2 + 3x + 1 \quad \underline{2}$$

$$18. 5x - 6 + 3x^2$$

$$3x^2 + 5x - 6 \quad \underline{3}$$

$$19. 9x^2 + 2 + x^3 + x$$

$$x^3 + 9x^2 + x + 2 \quad \underline{1}$$

$$20. -3 + 3x^3 - x^2 + 4x$$

$$3x^3 - x^2 + 4x - 3 \quad \underline{3}$$

$$21. x^2 + 3x^3 + 27 - x$$

$$3x^3 + x^2 - x + 27 \quad \underline{3}$$

$$22. 25 - x^3 + x$$

$$-x^3 + x + 25 \quad \underline{-1}$$

$$23. x - 3x^2 + 4 + 5x^3$$

$$5x^3 - 3x^2 + x + 4 \quad \underline{5}$$

$$24. x^2 + 64 - x + 7x^3$$

$$7x^3 + x^2 - x + 64 \quad \underline{7}$$