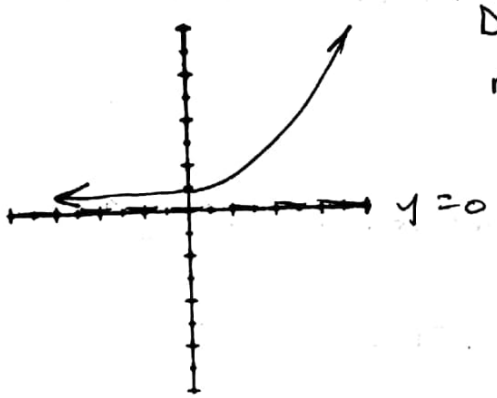
SEC 3.5 GEOMETRIC SEQUENCES

MAYO (F2017)

REVIEW

① GRAPH $y = 3^x$. STATE DOMAIN + RANGE + DESCRIBE END BEHAVIOR



D: $\mathbb{R}$ AS $x \to \infty$, $y \to \infty$

R: $y > 0$ AS $x \to -\infty$, $y \to 0$

② TELL WHETHER THE GRAPHS OF THE FOLLOWING EQUATIONS ARE PARALLEL, PERPENDICULAR, OR NEITHER

$y = -2x + 11$ AND $y + 2x = 23$

PARALLEL

$m = -2$ $y = -2x + 23$ $m = -2$

③ WRITE AN EQUATION FOR THE FUNCTION WHOSE VALUES ARE LISTED IN THE TABLE.

x	0	1	2	3	4	5
f(x)	0	3	6	9	12	15

(+1 between each x; +3 between each f(x))

$m = \frac{3}{1} = 3$

$b = 0$

$y = 3x$

ARITHMETIC SEQUENCES – PREVIOUSLY WE HAVE SEEN THAT A SEQUENCE OF NUMBERS WITH A COMMON DIFFERENCE (d) IS AN ARITHMETIC SEQUENCE.

$a_n = a_1 + (n-1)d$

ex 5, 9, 13, 17, ... (+4 +4 +4) $d = 4$

* THE nth TERM IS $a_n = a_1 + (n-1)d = 5 + (n-1)4$
* THE 25th TERM IS $a_{25} = a_{25} = 5 + (25-1)4 = 5 + (24)(4) = 5 + 96 = 101$
* WHAT TERM EQUALS 73?

$73 = 5 + (n-1)4$

$73 = 5 + 4n - 4 = 4n + 1$

$4n + 1 = 73$

$4n = 72$

$n = 72/4 = 18$th

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GEOMETRIC SEQUENCES

THE SEQUENCE 1, 5, 25, 125, ... IS AN EXAMPLE OF A GEOMETRIC SEQUENCE. INSTEAD OF A COMMON DIFFERENCE, IT HAS A COMMON RATIO. THE COMMON RATIO CAN BE FOUND BY DIVIDING A TERM BY THE PREVIOUS TERM.

EXAMPLE 1. DETERMINE WHETHER EACH SEQUENCE IS GEOMETRIC, ARITHMETIC, OR NEITHER.

(A) 256, 128, 64, 32 .... GEOMETRIC $r = \frac{1}{2}$

$\times \frac{1}{2}$ $\times \frac{1}{2}$ $\times \frac{1}{2}$

(B) 4, 9, 12, 18, ... NEITHER

+3 +3 +6

(C) 25, 21, 17, ... ARITHMETIC $d = -4$

-4 -4

APPLICATION 1. DETERMINE WHETHER EACH SEQUENCE IS GEOMETRIC, ARITHMETIC, OR NEITHER.

×3 ×3 ×3

(1) 1, 3, 9, 27 ... GEOMETRIC $r = 3$

(2) -20, -15, -10, -5, ... ARITHMETIC $d = 5$

+5 +5 +5

(3) 2, 8, 14, 22 ... NEITHER

+6 +6 +8

EXAMPLE 2. FIND THE NEXT 3 TERMS IN EACH SEQUENCE. GEOMETRIC

(A) 1, -4, 16, -64, ...

$r = \frac{-4}{1} = -4$

$(-4)(-64) = 256$ $256(-4) = -1004$

$-1004(-4) = 4016$

(B) 9, 3, 1, $\frac{1}{3}$, ...

$r = \frac{3}{9} = \frac{1}{3}$

$\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$

(C) 24, 36, 54, ...

$r = \frac{36}{24} = \frac{6}{4} = \frac{3}{2}$

$\frac{3}{2}(54) = 81$

$\frac{3}{2}(81) = 121.5$

$\frac{3}{2}(121.5) = 182.25$

(side work: 27×3; $243 \div 2 = 121.5$; $121.5 \times 3 = 364.5$; $364.5 \div 2 = 182.25$; (34))

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APPLICATION 2. FIND THE NEXT 3 TERMS OF EACH SEQUENCE.

(1) $-3, 15, -75, 375, \ldots$ $r = -5$ $\boxed{-1875, 9375, -46,875}$

(4) $15, 11, 7, 3, \ldots$ $\boxed{-1, -5, -9}$ $d = -4$

(5) $24, 36, 54, 81, \ldots$ $r = 1.5$ $\boxed{121.5, 182.25, 273.375}$

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* FORMULA FOR THE nth TERM *

JUST LIKE THE ARITHMETIC SEQUENCE, THERE IS A FORMULA FOR THE nth TERM OF A GEOMETRIC SEQUENCE, a_n.

(1*) SUPPOSE WE HAVE THE SEQUENCE $1, 2, 4, 8, \ldots$

WHAT IS THE COMMON RATIO, r? $r = 2$

WHAT DO YOU DO TO THE 1ST TERM a_1 TO GET a_2?

$2(a_1) = a_2$

WHAT DO YOU DO TO THE 2ND TERM a_2 TO GET a_3?

$2(a_2) = a_3 \quad 2(2a_1) = 4a_1 = a_3 \quad 2^2 a_1 \quad a_3 = 2^2 a_1$

$a_4 = 2^3 a_1$

$a_n = 2^{n-1} a_1$

HOW DO YOU GET THE nth TERM?

FORMULA FOR a_n: $\boxed{a_n = a_1 r^{n-1}}$

EXAMPLE 3. FIND THE FORMULA FOR THE nth TERM, a_n.

(A) $-6, 12, -24, 48, \ldots$ $r = -2$, $a_1 = -6$ $\boxed{a_n = -6(-2)^{n-1}}$

(B) $81, 9, 1, \frac{1}{9}, \ldots$ $a_1 = 81$, $r = \frac{1}{9}$ $\boxed{a_n = 81\left(\frac{1}{9}\right)^{n-1}}$

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APPLICATION 3. GIVE THE FORMULA FOR THE nth TERM, a_n.

(1) 96, 48, 24, 12, ... $a_1 = 96$, $r = \frac{1}{2}$ $\quad a_n = 96\left(\frac{1}{2}\right)^{n-1}$

(2) 212, 106, 53, ... $a_1 = 212$, $r = \frac{1}{2}$ $\quad a_n = 212\left(\frac{1}{2}\right)^{n-1}$

PRACTICE

(1) ARITHMETIC, GEOMETRIC, OR NEITHER?

(A) 4, 1, 2, ... NEITHER

(B) 10, 20, 30, 40, ... ARITH, $d = 10$ (+10, +10)

(C) 4, 20, 100, ... GEOM, $r = 5$ (×5, ×5)

(D) 212, 106, 53, ... GEOM, $r = 1/2$ (×1/2, ×1/2)

(E) −10, −8, −6, −4 ARITH, $d = +2$ (+2, +2, +2)

(F) 5, −10, 20, 40, ... NEITHER (−2, −2, 2)

(2) FIND THE NEXT 3 TERMS.

(A) 2, −10, 50, ... −250, 1250, −6250

(B) 36, 12, 4, ... $\frac{4}{3}, \frac{4}{9}, \frac{4}{27}$ (×1/3, ×1/3)

(C) 4, 12, 36, ... 108, 324, 972 (×3, ×3)

(D) 400, 100, 25, ... $\frac{25}{4}, \frac{25}{16}, \frac{25}{64}$ (×1/4, ×1/4)

(E) −6, −42, −294, ... −2058, −14406, −100842 (×7, ×7)

(F) 1024, −128, 16, ... $r = -\frac{1}{8}$ $\quad -2, \frac{1}{4}, -\frac{1}{32}$

(3) THE FIRST TERM OF A GEOMETRIC SEQUENCE IS 1 AND THE COMMON RATIO IS 9. WHAT IS THE 8th TERM, a_8? $a_1 = 1$ $r = 9$

$$a_8 = 1(9)^7 = \boxed{4{,}782{,}969}$$

(4) $a_1 = 2$, $r = 4$. WHAT IS THE 14th TERM, a_{14}?

$$a_{14} = 2(14)^{13} = \boxed{1.58 \times 10^{15}}$$

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NAME _____ DATE _____ PERIOD _____

Practice

Geometric Sequences as Exponential Functions

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

1. 1, −5, −11, −17, ... ARITH. $d = -6$

2. $3, \frac{3}{2}, 1, \frac{3}{4}, \ldots$ NEITHER

3. 108, 36, 12, 4, ... GEOM $r = \frac{1}{3}$

4. −2, 4, −6, 8, ... NEITHER.

Find the next three terms in each geometric sequence.

5. 64, 16, 4, ... $r = \frac{1}{4}$ $1, \frac{1}{4}, \frac{1}{16}$

6. 2, −12, 72, ... $r = -6$ $-432, 2592, -15552$

7. 3750, 750, 150, ... $30, 6, \frac{6}{5}$ $r = \frac{1}{5}$

8. 4, 28, 196, ... $r = 7$ $1372, 9604, 67228$

9. Write an equation for the nth term of the geometric sequence 896, −448, 224, ... Find the eighth term of this sequence.

$a_1 = 896$ $r = -\frac{1}{2}$ $a_n = 896\left(\frac{1}{2}\right)^{n-1}$ $a_8 = 896\left(\frac{1}{2}\right)^7 = 7$

10. Write an equation for the nth term of the geometric sequence 3584, 896, 224, ... Find the sixth term of this sequence.

$a_1 = 3584$ $r = \frac{1}{4}$ $a_n = 3584\left(\frac{1}{4}\right)^{n-1}$ $a_6 = 3584\left(\frac{1}{4}\right)^5 = 3.5$

11. Find the sixth term of a geometric sequence for which $a_2 = 288$ and $r = \frac{1}{4}$.

$a_1 = 4a_2 = 4(288) = 1152$

$a_6 = 1152\left(\frac{1}{4}\right)^5 = 1.125$

12. Find the eighth term of a geometric sequence for which $a_3 = 35$ and $r = 7$.

$a_2 = \frac{1}{7}(35) = 5$

$a_1 = \frac{1}{7}(5) = \frac{5}{7}$

$a_8 = \frac{5}{7}(7)^7 = 588{,}245$

13. **PENNIES** Thomas is saving pennies in a jar. The first day he saves 3 pennies, the second day 12 pennies, the third day 48 pennies, and so on. How many pennies does Thomas save on the eighth day?

3, 12, 48 $a_1 = 3$ $r = 4$ $a_8 = 3(4)^7$

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