SEC 3.4 - GROWTH + DECAY

MAYO (F2017)

REVIEW:

① THE TABLE SHOWS THE RELATIONSHIP BETWEEN CALORIES + GRAMS OF FAT IN AN ORDER OF FRIED CHICKEN FROM VARIOUS RESTAURANTS.

$\hat{y} = 0.707726027x + 5.855013699$

CALORIES	305	410	320	500	510	440
FAT (g)	28	34	28	41	42	38

ASSUMING A LINEAR MODEL, HOW MANY GRAMS OF FAT WOULD YOU EXPECT TO BE IN AN ORDER WITH 275 CALORIES?

Ⓐ 22 Ⓑ 25 Ⓒ 28 Ⓓ 30

$y_1(275) \approx 25.3$

② SIMPLIFY:

Ⓐ $\sqrt{48}$

$2 \quad 24$, $2 \quad 12$, $2 \quad 6$, $2 \quad 3$

$4\sqrt{3}$

Ⓑ $\frac{r^8 t^{12}}{r^2 t^7}$

$r^6 t^5$

Ⓒ $\left(\frac{4n^2p^4}{8p^3}\right)^3 = \frac{4^3 n^6 p^{12}}{8^3 p^9}$

$\frac{64}{512} = \frac{1}{8}$

$= \frac{n^6 p^3}{8}$

*** EQUATION FOR EXPONENTIAL GROWTH ***

$$y = a(1+r)^t$$

t ← TIME

r ← RATE OF CHANGE (DECIMAL)

a ← INITIAL AMOUNT

y ← FINAL AMOUNT

EXAMPLE 1. THE PRIZE FOR A RADIO CONTEST BEGINS WITH \$100. ONCE A DAY, A NAME IS CHOSEN. THE PERSON HAS 15 MINUTES TO CALL OR THE PRIZE INCREASES 2.5%.

Ⓐ WRITE AN EQUATION TO REPRESENT THE AMOUNT AFTER t DAYS.

$y = a(1+r)^t$

$y = 100(1+0.025)^t$

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(B) HOW MUCH WILL THE PRIZE BE WORTH AFTER 10 DAYS?

$$y = 100(1 + 0.025)^{10}$$

$$\approx 128.01 \Rightarrow \$128.01$$

APPLICATION 1. A COLLEGE'S TUITION HAS RISEN 5% EACH YEAR SINCE 2000. IF THE TUITION IN 2000 WAS $10,850, WRITE AN EQUATION FOR THE AMOUNT OF TUITION t YEARS AFTER 2000. PREDICT THE COST FOR 2015. SHOW WORK.

$$y = 10{,}850(1 + 0.05)^t$$

$$y(15) = 10{,}850(1 + 0.05)^{15} \approx \boxed{\$22556.37}$$

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* EQUATION FOR COMPOUND INTEREST *

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

n = # OF TIMES COMPOUNDED PER YEAR

t = # OF YRS.

r IS ANNUAL INTEREST RATE (DECIMAL)

PRINCIPAL (INITIAL AM'T)

CURRENT AMOUNT

EXAMPLE 2. MARIA'S PARENTS INVESTED $14,000 AT 6% COMPOUNDED MONTHLY. HOW MUCH MONEY WILL THEY HAVE AFTER 10 YEARS?

$$A = 14{,}000\left(1 + \frac{0.06}{12}\right)^{12t}$$

$$A(10) = 14{,}000\left(1 + \frac{0.06}{12}\right)^{12(10)} \approx \boxed{\$25471.55}$$

APPLICATION 2. DETERMINE THE AMOUNT OF AN INVESTMENT IF $300 IS INVESTED AT AN INTEREST RATE OF 3.5% COMPOUNDED MONTHLY FOR 22 YRS. SHOW SET-UP.

$$A = 300\left(1 + \frac{0.035}{12}\right)^{12t}$$

$$A(22) = 300\left(1 + \frac{0.035}{12}\right)^{(12)(22)} \approx \boxed{\$647.20}$$

* EQUATION FOR EXPONENTIAL DECAY *

$$y = a(1-r)^t$$

TIME. DECAY RATE (DECIMAL). FINAL AMT. INITIAL AM'T.

EXAMPLE 3.

A FULLY INFLATED CHILD'S RAFT FOR A POOL IS LOSING 6.6% OF ITS AIR EVERY DAY. THE RAFT ORIGINALLY CONTAINED 4500 in^3 OF AIR.

(A) WRITE AN EQUATION TO REPRESENT THE LOSS OF AIR.

$$y = 4500(1-0.066)^t$$

(B) ESTIMATE THE AMOUNT OF AIR LEFT AFTER 7 DAYS. SHOW SET-UP.

$$y(7) = 4500(1-0.066)^7 \approx \boxed{2790 \text{ in}^3}$$

(C) ESTIMATE HOW MANY DAYS UNTIL THE RAFT CONTAINS 2500 in^3 OF AIR. SHOW SET-UP.

$Y_1 = 4500(1-0.066)^x$ $\quad$ $2500 = 4500(1-0.066)^x$ $\quad$ TBL

8 2606.1
9 2434.1 } DURING 8th DAY.

APPLICATION 3. THE POPULATION OF CAMPBELL COUNTY, KY HAS BEEN DECREASING AT AN AVERAGE RATE OF 0.3% PER YEAR. IN 2000, ITS POPULATION WAS 88,647.

(A) WRITE AN EQUATION FOR THIS SITUATION.

$$y = 88{,}647(1-0.003)^t$$

(B) PREDICT THE POPULATION IN 2010. SHOW SET-UP.

$$y(10) = 88647(1-0.003)^{10} \approx \boxed{85642 \text{ PEOPLE}}$$

(C) PREDICT WHAT YEAR THE POPULATION WILL BE AT 80,000. SHOW SET-UP.

$$80{,}000 = 88647(1-0.003)^t$$

TBL

5 - 82232
6 - 79559 } DURING 5th YR.

PRACTICE.

(1) MS. ACOSTA RECEIVED A JOB AS A TEACHER WITH A STARTING SALARY OF \$34,000. ACCORDING TO HER CONTRACT, SHE WILL RECEIVE A 1.5% INCREASE IN HER SALARY EVERY YEAR. HOW MUCH WILL MS. ACOSTA EARN IN 7 YEARS? SHOW SET-UP.

$y = 34000(1+0.015)^t \quad y(7) = 34000(1+0.015)^7 \approx$ \$37734.73

(2) PAUL INVESTED \$400 IN AN ACCOUNT WITH 5.5% INTEREST COMPOUNDED MONTHLY. HOW MUCH WILL HIS INVESTMENT BE WORTH IN 8 YEARS? SHOW SET-UP.

$A = 400(1+0.055)^t \quad A(8) = 400(1+0.055)^8 \approx \boxed{\$613.87}$

(3) IN 2000, 2200 STUDENTS ATTENDED POLARIS H.S. THE ENROLLMENT HAS BEEN DECLINING 2% ANNUALLY.

(A) WRITE AN EQUATION FOR THE ENROLLMENT t YEARS AFTER 2000.

$y = 2200(1-0.02)^t$

(B) HOW MANY STUDENTS WILL BE ENROLLED IN 2015? SHOW SET-UP.

$y(15) = 2200(1-0.02)^{15} \approx$ 1625 STUDENTS

(C) WHAT YEAR WILL THE ENROLLMENT BE 2100 STUDENTS? SHOW SET UP.

TBL

2 - 2112.9
3 - 2070.6 } DURING 2ND YR

(4) THE NUMBER OF PEOPLE WHO OWN COMPUTERS HAS INCREASED 23.2% ANNUALLY SINCE 1990. IF HALF A MILLION PEOPLE OWNED A COMPUTER IN 1990, PREDICT HOW MANY PEOPLE WILL OWN A COMPUTER IN 2018. SHOW SET-UP.

$y = 500{,}000(1+.232)^t$

$$\begin{array}{r} 2018 \\ -1990 \\ \hline 28 \end{array}$$

$y(28) = 500{,}000(1+.232)^{28}$

$\approx \boxed{172{,}214{,}543 \text{ PEOPLE}}$

(5) GIVEN THE EQUATION $A = 200(1+0.05)^t$, WHAT DOES:

(A) 200 REPRESENT? INITIAL VALUE

(B) 0.05 REPRESENT? GROWTH RATE

(C) t REPRESENT? TIME

(D) A REPRESENT? FINAL AMOUNT

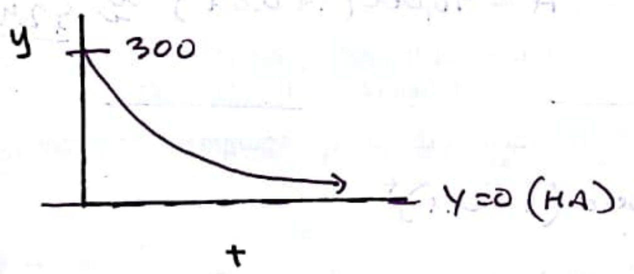
(6) GIVEN THE EQUATION $y = 300(0.88)^t$,

(A) IS THIS GROWTH OR DECAY? DECAY.

(B) HOW MUCH (%) IS BEING LOST OR GAINED FOR EACH TIME PERIOD? 12% LOST.

(C) WHAT DOES 300 REPRESENT? INITIAL VALUE.

(D) SKETCH A POSSIBLE GRAPH FOR THIS FUNCTION. LABEL APPROPRIATE POINTS AND THE HORIZONTAL ASYMPTOTE.



(E) WHAT ARE THE DOMAIN AND RANGE FOR THIS FUNCTION (ASSUMING TIME ≥ 0)?

D: $x \geq 0$ R: $0 < y \leq 300$

(7) GIVEN THE EQUATION $A = 3000\left(1+\frac{0.05}{4}\right)^{4t}$,

(A) WHAT IS THE INITIAL AMOUNT? 3000

(B) HOW OFTEN WILL THE INVESTMENT BE COMPOUNDED?

QUARTERLY (4 TIMES / YEAR)

7-6 Skills Practice

Growth and Decay

1. **POPULATION** The population of New York City increased from 8,008,278 in 2000 to 8,168,388 in 2005. The annual rate of population increase for the period was about 0.4%. $\Rightarrow 0.004$

   a. Write an equation for the population t years after 2000.

   ~~$y = 8{,}008{,}278(1+0.004)^t$~~ $y = 8{,}008{,}278(1+.004)^t$

   b. Use the equation to predict the population of New York City in 2015.

   $y(15) = 8{,}008{,}278(1+0.004)^{15} \approx$ 8,502,469 people

2. **SAVINGS** The Fresh and Green Company has a savings plan for its employees. If an employee makes an initial contribution of $1000, the company pays 8% interest compounded quarterly.

   a. If an employee participating in the plan withdraws the balance of the account after 5 years, how much will be in the account?

   $A = 1000\left(1+\frac{0.08}{4}\right)^{4t}$ $A(5) = 1000\left(1+\frac{0.08}{4}\right)^{4(5)} \approx$ \$1485.95

   b. If an employee participating in the plan withdraws the balance of the account after 35 years, how much will be in the account?

   $A(35) = 1000\left(1+\frac{0.08}{4}\right)^{4(35)} \approx \boxed{\$15{,}996.47}$

3. **HOUSING** Mr. and Mrs. Boyce bought a house for $96,000 in 1995. The real estate broker indicated that houses in their area were appreciating at an average annual rate of 7%. If the appreciation remained steady at this rate, what was the value of the Boyce's home in 2009?

   $2009 - 1995 = 14$

   $A = 96{,}000(1+0.07)^{14} \approx \boxed{\$247{,}539.28}$

4. **MANUFACTURING** Zeller Industries bought a piece of weaving equipment for $60,000. It is expected to depreciate at an average rate of 10% per year.

   a. Write an equation for the value of the piece of equipment after t years.

   $y = 60{,}000(1-0.1)^t$

   b. Find the value of the piece of equipment after 6 years.

   $y(6) = 60{,}000(1-0.1)^6 \approx$ \$31,886.46

5. **FINANCES** Kyle saved $500 from a summer job. He plans to spend 10% of his savings each week on various forms of entertainment. At this rate, how much will Kyle have left after 15 weeks?

   $y = 500(1-0.1)^{15} \approx$ \$102.95

6. **TRANSPORTATION** Tiffany's mother bought a car for $9000 five years ago. She wants to sell it to Tiffany based on a 15% annual rate of depreciation. At this rate, how much will Tiffany pay for the car?

   $y = 9000(1-0.15)^5 \approx$ \$3993.35

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