

SEC 3.1 : PROPERTIES OF EXPONENTS

AN EXPONENTIAL EXPRESSION INCLUDES A BASE RAISED TO AN EXPONENT OR POWER. THE PROPERTIES OF EXPONENTS HELP US SIMPLIFY EXPONENTIAL EXPRESSIONS.

① PRODUCT OF POWERS

$$a^n \cdot a^m = a^{n+m}$$

$$\text{ex) } 2x^4 \cdot 3x^3 = 6x^{4+3} = 6x^7$$

APPLICATION 1. SIMPLIFY.

$$\text{① } (3a)(3a^2) = 9a^3$$

$$\text{② } (ab^2)(2a^2b) = 2a^3b^3$$

$$\text{③ } (2x^4y)(3xy^2) = 6x^5y^3$$

② POWER OF A PRODUCT

$$(ab)^m = a^m b^m$$

$$\text{ex) } (3xy)^4 = 3^4 x^4 y^4 = 81x^4y^4$$

APPLICATION 2. SIMPLIFY.

$$\text{④ } (xy)^2 = x^2y^2$$

$$\text{⑤ } (3xy)^4 = 3^4 x^4 y^4 = 81x^4y^4$$

$$\text{⑥ } (4x)^2 = 4^2 x^2 = 16x^2$$

③ POWER OF A POWER

$$(a^n)^m = a^{n \cdot m}$$

$$\text{ex) } (x^4)^2 = x^{4 \cdot 2} = x^8$$

APPLICATION 3. SIMPLIFY.

$$\text{⑦ } (w^3)^5 = w^{15}$$

$$\text{⑧ } (2w^2)^3 = 2^3 w^6 = 8w^6$$

$$\text{⑨ } (3w^2y^7)^2 = 3^2 w^4 y^{14} = 9w^4y^{14}$$

PRACTICE. SIMPLIFY

① $x^2 \cdot x^3 = x^5$

⑥ $(2yk^2)^3 = 8y^3k^6$

② $(3x^2)^3 = 27x^6$

⑦ $2x(12x^2y) = 24x^3y$

③ $(2y^4)^3 = 8y^{12}$

⑧ $(3x^2)(2xy) = 6x^3y$

④ $2y^2 \cdot 2y^3 = 4y^5$

⑨ $(-5z^3)(-2z^2) = 10z^5$

⑤ $(3x)(3x) = 9x^2$

⑩ $(4a)^3 = 4^3a^3 = 64a^3$

④ POWER OF ZERO

$a^0 = 1$

ex) $(25x)^0 = 1$

APPLICATION 4. SIMPLIFY.

⑩ $4^0 = 1$

⑪ $3y^0 = 3(1) = 3$ ⑫ $3(2)^0 = 3(1) = 3$

⑤ QUOTIENT OF POWERS

$\frac{a^n}{a^m} = a^{n-m}$

ex) $\frac{6x^8}{2x^4} = 3x^{8-4} = 3x^4$

APPLICATION 5. SIMPLIFY.

⑬ $\frac{6c^8}{2c^4} = 3c^4$

⑭ $\frac{12x^2y^3}{6x^2y} = 2y^2$

CROSS OUT METHOD

⑮ $\frac{x^2yz^3}{xy z^3} = x$

⑯ $\frac{8n^5}{4n^2} = 2n^3$

PRACTICE. SIMPLIFY.

① $\left(\frac{3x^2y^7}{21xy^6}\right)^0 = 1$

② $x^2 \cdot 3x = 3x^3$

③ $(2x^2)^3 = 2^3x^6 = 8x^6$

④ $(pq^5)^2 = p^2q^{10}$

②

CONTINUED.
→

$$(5) (g^7)(h^7) = (g^7 h^7)$$

$$(6) (n^6 m^2)^0 = 1$$

$$(7) (x^2)^2 (2xy^2)^3 = (x^4)(8x^3y^6) = 8x^7y^6$$

$$(8) \frac{12x^5y^2z^6}{4xy} = 3x^4yz^6$$

6 POWER OF A QUOTIENT

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$(9) \left(\frac{3x}{y^2}\right)^2 = \frac{3^2x^2}{y^4} = \frac{9x^2}{y^4}$$

APPLICATION 6. SIMPLIFY.

$$(17) \left(\frac{x^2}{y^3}\right)^5 = \frac{x^{10}}{y^{15}}$$

$$(18) \left(\frac{2x}{y^2}\right)^2 = \frac{4x^2}{y^4}$$

$$(19) \left(\frac{2}{x^2}\right)^3 = \frac{8}{x^6}$$

$$(20) \left(\frac{2s}{r}\right)^4 = \frac{16s^4}{r^4}$$

7 NEGATIVE POWERS

$$a^{-n} = \frac{1}{a^n} \quad \frac{1}{a^{-n}} = a^n$$

$$(21) x^{-6} = \frac{1}{x^6}$$

$$\text{ADD } \frac{1}{3x^{-2}} = \frac{x^2}{3}$$

NO NEGATIVE EXPONENTS!

APPLICATION 7. SIMPLIFY.

$$(21) x^{-7} = \frac{1}{x^7}$$

$$(22) \frac{1}{2x^{-7}} = \frac{x^7}{2}$$

$$(23) (x^2y^{-5})^2 = x^4y^{-10} = \frac{x^4}{y^{10}}$$

$$(24) (x^{-4})(x^4) = x^{-4+4} = x^0 = 1$$

$$(25) (-2x)^{-2} =$$

$$(26) \frac{y^5}{-16x^2y^2} = \frac{y^3}{-16x^2}$$

$$\frac{1}{(-2x)^2} = \frac{1}{4x^2}$$

PROPERTIES

① PRODUCT OF POWERS

$$a^n \cdot a^m = a^{n+m}$$

② POWER OF A PRODUCT

$$(ab)^m = a^m b^m$$

③ POWER OF A POWER

$$(a^n)^m = a^{n \cdot m}$$

④ POWER OF ZERO

$$a^0 = 1$$

⑤ QUOTIENT OF POWERS

$$\frac{a^n}{a^m} = a^{n-m}$$

⑥ POWER OF A QUOTIENT

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

⑦ NEGATIVE POWERS

$$a^{-n} = \frac{1}{a^n} \quad \frac{1}{a^{-n}} = a^n$$

EXAMPLES

$$2x^4 \cdot 3x^3 = 6x^{4+3} = 6x^7$$

$$(3xy)^4 = 3^4 x^4 y^4 = 81x^4 y^4$$

$$(w^5)^2 = w^{5 \cdot 2} = w^{10}$$

$$(25k)^0 = 1$$

$$\frac{6c^8}{2c^4} = 3c^{8-4} = 3c^4$$

$$\left(\frac{2s}{r}\right)^4 = \frac{2^4 s^4}{r^4} = \frac{16s^4}{r^4}$$

$$w^{-6} = \frac{1}{w^6} \quad \frac{1}{y^{-5}} = y^5$$

PRACTICE. SIMPLIFY.

$$\textcircled{1} (2m^{-2}n^6)(-2m^{-2}n)^3 = (2m^{-2}n^6)(-8m^{-6}n^3) = -16m^{-8}n^9 = \frac{-16n^9}{m^8}$$

$$\textcircled{2} 4x^5 \cdot \left(\frac{3x^{-3}}{x^7}\right)^2 = 4x^5 \left(\frac{9x^{-6}}{x^{14}}\right) = \frac{36x^5 x^{-6}}{x^{14}} =$$

$$\frac{36x^{-1}}{x^{14}} = \frac{36}{x^1 \cdot x^{14}} = \frac{36}{x^{15}}$$

④

Properties of Exponents

Simplify. Your answer should contain only positive exponents.

1) $2m^2 \cdot 2m^3$

$4m^5$

2) $m^4 \cdot 2m^{-3}$

$2m$

3) $4r^{-3} \cdot 2r^2$

$8r^{-1} = \frac{8}{r}$

4) $4n^4 \cdot 2n^{-3}$

$8n$

5) $2k^4 \cdot 4k$

$8k^5$

6) $2x^3y^{-3} \cdot 2x^{-1}y^3$

$4x^2y^0 = 4x^2$

7) $2y^2 \cdot 3x$

$6xy^2$

8) $4v^3 \cdot vu^2$

$4v^4u^2$

9) $4a^3b^2 \cdot 3a^{-4}b^{-3}$

$= 12a^{-1}b^{-1} = \frac{12}{ab}$

10) $x^2y^{-4} \cdot x^3y^2$

$x^5y^{-2} = \frac{x^5}{y^2}$

11) $(x^2)^0$

$= 1$

12) $(2x^2)^{-4}$

$= 2^{-4}x^{-8} = \frac{1}{16x^8}$

13) $(4r^0)^4$

$= (4)^4 = 256$

14) $(4a^3)^2$

$= 4^2a^6 = 16a^6$

15) $(3k^4)^4$

$= 3^4k^{16} = 81k^{16}$

16) $(4xy)^{-1}$

$= \frac{1}{4xy}$

$$17) (2b^4)^{-1} = \frac{1}{2b^4}$$

$$18) (x^2y^{-1})^2 = x^4y^{-2} = \frac{x^4}{y^2}$$

$$19) (2x^4y^{-3})^{-1} = \frac{1}{2x^4y^{-3}} = \frac{y^3}{2x^4}$$

$$20) (3m)^{-2} = 3^{-2}m^{-2} = \frac{1}{9m^2}$$

$$21) \frac{r^2}{2r^3} = \frac{1}{2r}$$

$$22) \frac{x^{-1}}{4x^4} = \frac{1}{4x^5}$$

$$23) \frac{3n^4}{3n^3} = n$$

$$24) \frac{m^4}{2m^4} = \frac{1}{2}$$

$$25) \frac{3m^{-4}}{m^3} = \frac{3}{m^3 \cdot m^4} = \frac{3}{m^7}$$

$$26) \frac{2x^4y^{-4}z^{-3}}{3x^2y^{-3}z^4} = \frac{2x^4y^3}{3x^2y^4z^3z^4} = \frac{2x^2}{yz^7}$$

$$27) \frac{4x^0y^{-2}z^3}{4x} = \frac{4z^3}{4xy^2} = \frac{z^3}{xyz}$$

$$28) \frac{2h^3j^{-3}k^4}{3jk} = \frac{2h^3k^4}{3jj^3k} = \frac{2h^3k^3}{3j^4}$$

$$29) \frac{4m^4n^3p^3}{3m^2n^2p^4} = \frac{4m^2n}{3p}$$

$$30) \frac{3x^3y^{-1}z^{-1}}{x^{-4}y^0z^0} = \frac{3x^3x^4}{yz} = \frac{3x^7}{yz}$$

6