

Section 2.6: Arithmetic Sequences

MAYO 2017

NC.M1.F-BF.1 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

NC.M1.F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading from a table).

Review. Complete the following. Show work!

1. If $f(x) = 3x + 2$, find:

a. $f(2) = 3(2) + 2$

$= 6 + 2 = 8$

b. $f(-1) =$

$3(-1) + 2 =$

$-3 + 2 = -1$

c. x if $f(x) = 29$

$29 = 3x + 2$

$-2 \quad -2$
 $\hline 27 = 3x \quad x = 9$

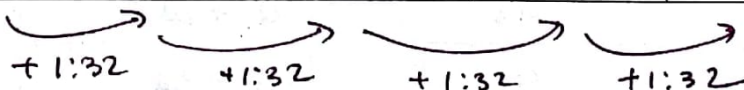
2. Compute the slope of the line parallel to the line that passes through the points $(-2, 5)$ and $(3, -5)$.

$m = \frac{5 - (-5)}{-2 - 3} = \frac{10}{-5} = -2$

A sequence is a SET OF NUMBERS. The numbers themselves are called the TERMS OF THE SEQUENCE. Often times when we are given a sequence we will be asked to find a pattern in it.

Example 1. Given the table of distance and times, is there a pattern?

Distance (m)	400	800	1200	1600	2000
Time (min:sec)	1:32	3:04	4:36	6:08	7:40



An **Arithmetic Sequence** is a numerical pattern that INCREASES OR DECREASES at a CONSTANT RATE called the COMMON DIFFERENCE. (d)

Example 2: 3, 5, 7, 9, 11, ...

$+2 \quad +2 \quad +2$

$d = 2$

Example 3: 33, 29, 25, 21, 17, ...

$-4 \quad -4 \quad -4 \quad -4$

$d = -4$

Application 1. Determine whether each sequence is an arithmetic sequence. Explain.

a. $-4, -2, 0, 2, \dots$
 $\begin{matrix} \rightarrow & \rightarrow & \rightarrow \\ +2 & +2 & +2 \end{matrix}$ $d=2$ ✓

b. $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{13}{16}, \dots$
 $\begin{matrix} \rightarrow & \rightarrow & \rightarrow \\ +\frac{1}{8} & +\frac{1}{8} & +\frac{1}{8} \end{matrix}$ ✓ $d = \frac{1}{8}$

c. $-26, -22, -18, -14, \dots$
 $\begin{matrix} \rightarrow & \rightarrow & \rightarrow \\ +4 & +4 & +4 \end{matrix}$ ✓ $d=4$

d. $1, 4, 9, 25, \dots$
 $\begin{matrix} \rightarrow & \rightarrow \\ 3 & 5 & 16 \end{matrix}$ NO

In an arithmetic sequence, you can use the common difference to find the next term.

Example 4. Find the next three terms of the arithmetic sequence $15, 9, 3, -3, \dots$
 $d = -6 \Rightarrow -9, -15, -21$
 $\begin{matrix} -9 & -15 & -21 \\ \leftarrow -6 & \leftarrow -6 & \leftarrow -6 \end{matrix}$

Application 2. Find the next four terms of the arithmetic sequence $9.5, 11.0, 12.5, 14.0, \dots$

$d = 1.5$
 $14 + 1.5 = 15.5$
 $15.5 + 1.5 = 17$
 $16 + 1.5 = 17.5$
 $17.5 + 1.5 = 19$
 $15.5, 16, 17.5, 19$

Each term of an arithmetic sequence can be expressed in terms of the first term a_1 and the common difference d . For the table below, use the sequence: $8, 11, 14, \dots$

$a_1 = 8$ $d = 3$

Term	Symbol	In terms of a_1 and d	Numbers
First term	a_1	a_1	8
Second term	a_2	$a_1 + d$	$8 + 1(3) = 11$
Third term	a_3	$a_1 + 2d$	$8 + 2(3) = 14$
Fourth term	a_4	$a_1 + 3d$	$8 + 3(3) = 17$
nth term	a_n	$a_1 + (n-1)d$	$8 + (n-1)3$

The nth Term of an Arithmetic Sequence

The nth term of an arithmetic sequence with first term a_1 and common difference d is given by

$a_n = a_1 + (n-1)d$

Example 5.

a. Write an equation for the nth term of the arithmetic sequence $-12, -8, -4, 0, \dots$

① FIND COMMON DIFF: $d = +4$

② WRITE EQUATION: $a_n = a_1 + (n-1)d$ $a_n = -12 + (n-1)4$

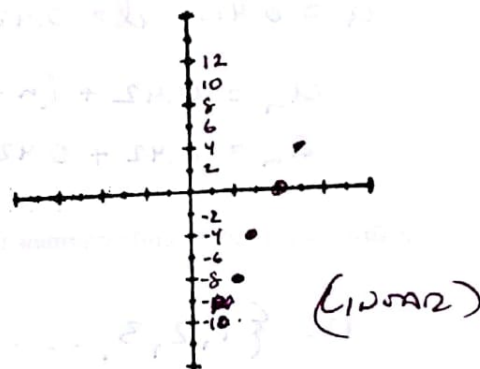
b. Find the 9th term of the sequence.

$a_9 = 4(9) - 16 = 36 - 16 = 20$

$a_n = -12 + 4n - 4$
 $a_n = 4n - 16$

c. Graph the first five terms of the sequence.

n	$4n - 16$	a_n	(n, a_n)
1	$4(1) - 16$	-12	$(1, -12)$
2	$4(2) - 16$	-8	$(2, -8)$
3	$4(3) - 16$	-4	$(3, -4)$
4	$4(4) - 16$	0	$(4, 0)$
5	$4(5) - 16$	4	$(5, 4)$



d. Which term of the sequence is 32?

$32 = 4n - 16$
 $+16 \quad +16$

$\frac{48}{4} = \frac{4n}{4} \quad n = 12 \Rightarrow 12\text{th Term } a_{12} = 32$

Application 3. Consider the sequence: $3, -10, -23, -36, \dots$

a. Write the equation for the nth term of the sequence.

$a_1 = 3 \quad d = -13$

$a_n = 3 + (n-1)(-13)$

$a_n = 3 - 13n + 13$

$a_n = -13n + 16$

b. Find the 15th term of the sequence.

$a_{15} = -13(15) + 16 = -195 + 16 = -179$

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c. Which term of the sequence is -114?

$-114 = -13n + 16$
 $-16 \quad -16$
 $-130 = -13n$
 $n = 10$
 $a_{10} = -114$

Arithmetic Sequences and Functions. As you can see in the previous example the graph of the first five terms of the arithmetic sequence lie on a line. An arithmetic sequence is a linear function in which n is the independent variable, a_n is the dependent variable and d is the rate of change. The formula can be written as a function:

$$f(n) = (n-1)d + a_1 \text{ or } a_1 + (n-1)d$$

where n is a counting number.

Example 6. Marisol is mailing invitations to her birthday party. The arithmetic sequence 0.42, 0.84, 1.26, 1.68, ... represents the cost of postage.

a. Write a function to represent the sequence.

$$a_1 = 0.42 \quad d = 0.42$$

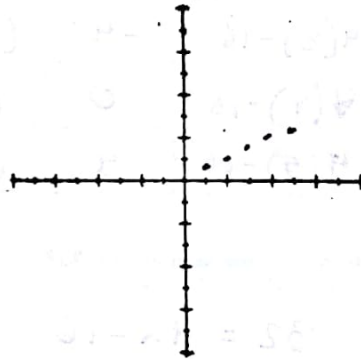
$$a_n = 0.42 + (n-1)(0.42)$$

$$a_n = 0.42 + 0.42n - 0.42$$

$$a_n = 0.42n \Rightarrow f(n) = 0.42n$$

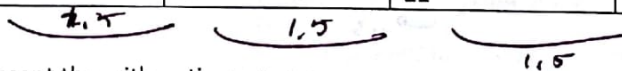
b. Graph the function and determine the domain.

$$D: \{1, 2, 3, \dots\}$$



Application 4. The chart below shows the length of Martin's long jumps.

Jump	1	2	3	4
Length (ft)	8	9.5	11	12.5



a. Write a function to represent the arithmetic sequence.

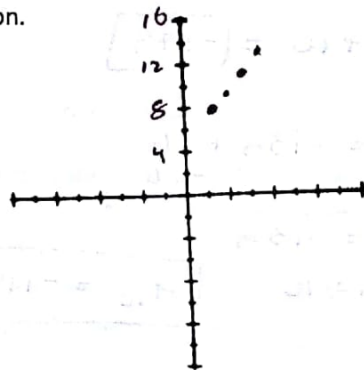
$$a_1 = 8 \quad d = 1.5$$

$$a_n = 8 + (n-1)(1.5)$$

$$a_n = 8 + 1.5n - 1.5$$

$$a_n = 6.5 + 1.5n$$

b. Graph the function.



$$f(n) = 1.5n + 6.5$$

Practice.

1. Determine whether each sequence is an arithmetic sequence.

a. 18, 16, 15, 13, ...
 $\begin{matrix} -2 & -1 & -2 \\ \hline \end{matrix}$ NO

b. 4, 9, 14, 19, ... \checkmark $d=5$
 $\begin{matrix} +5 & +5 & +5 \\ \hline \end{matrix}$

c. -10, -7, -4, 1, ... NO
 $\begin{matrix} +3 & +3 & +5 \\ \hline \end{matrix}$

d. -12.3, -9.7, -7.1, -4.5, ... \checkmark $d=2.6$
 $\begin{matrix} +2.6 & +2.6 & +2.6 \\ \hline \end{matrix}$ $\begin{matrix} 12.3 & 9.7 & 7.1 \\ 9.7 & 7.1 & 4.5 \\ \hline 2.6 & 2.6 & 2.6 \end{matrix}$

2. Find the next 3 terms of each arithmetic sequence.

a. 12, 9, 6, 3, ... 0, -3, -6
 $\begin{matrix} -3 & -3 & -3 \\ \hline \end{matrix}$

b. -2, 2, 6, 10, ... 14, 18, 22
 $\begin{matrix} +4 & +4 & +4 \\ \hline \end{matrix}$

c. 6, 12, 18, 24, ... 30, 36, 42
 $\begin{matrix} +6 & +6 & +6 \\ \hline \end{matrix}$

d. 0.02, 1.08, 2.14, 3.2, ... 4.26, 5.32, 6.38
 $1.06 = d$

3. Write an equation for the nth term of each arithmetic sequence.

a. 15, 13, 11, 9, ...
 $a_1 = 15$ $d = -2$
 $a_n = 15 + (n-1)(-2)$
 $a_n = -2n + 17$

b. -1, -0.5, 0, 0.5, ...
 $a_1 = -1$ $d = 0.5$
 $a_n = -1 + (n-1)(0.5) = -1 + 0.5n - 0.5$
 $a_n = 0.5n - 1.5$

c. -3, -8, -13, -18, ...
 $a_1 = -3$ $d = -5$
 $a_n = -3 + (n-1)(-5)$
 $a_n = -5n + 2$

d. -2, 3, 8, 13, ...
 $a_1 = 2$ $d = 5$
 $a_n = 2 + (n-1)(5) = 2 + 5n - 5$
 $a_n = 5n - 3$

4. Shiloh and her friends spent the day at an amusement park. In the first hour, they rode two rides. After 2 hours they had ridden 4 rides. They had ridden 6 rides after 3 hours. Write a function to represent the arithmetic sequence.

HR	1	2	3
# rides	2	4	6

$\begin{matrix} \searrow & \searrow \\ 2 & 2 \end{matrix}$

$f(n) = 2 + (n-1)2$
 $f(n) = 2 + 2n - 2$
 $f(n) = 2n$

5. The table shows how Ryan is paid at his lumber yard job.

Linear feet of 2x4 planks cut	10	20	30	40	50	69	70
Amount of money \$	8	16	24	32	40	48	56

$+8$

Write a function to represent Ryan's commission.

$f(n) = 10 + (n-1)8$ $f(n) = 10 + 8n - 8$
 $f(n) = 8n + 2$