

Section 1.4: More Words and Equations

NC.M1.A-CED.1 Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

NC.M1.A-CED.4 Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.

Review. Solve each equation or inequality. Show all steps.

1. $-5(6x + 6) = 30$

$$\begin{array}{r} -30x - 30 = 30 \\ +30 \quad +30 \end{array}$$

$$\frac{-30x}{-30} = \frac{60}{-30} \quad \boxed{x = -2}$$

2. $11 - 2(2x + 3) = 5 - 4x$

$$11 - 4x - 6 = 5 - 4x$$

$$5 - 4x = 5 - 4x$$

INFINITE SOLUTIONS

3. $-\frac{3}{5}a + 3 < -6$

$$5(-\frac{3}{5}a + 3 < -6)$$

$$-3a + 15 < -6$$

$$\begin{array}{r} -3a + 15 < -6 \\ -15 \quad -15 \end{array}$$

$$\frac{-3a}{-3} < \frac{-21}{-3}$$

$$\boxed{a > 7}$$

Often times in mathematics or science we are provided with formulas containing several variables. In many cases we will need to solve for a given variable of interest. We will use the same techniques we have recently reviewed for solving equations and inequalities.

Example 1. The formula for the circumference of a circle is $C = 2\pi r$ where C = circumference and r = length of the radius. Solve the formula for r .

$$C = 2\pi r = (2\pi) r$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \quad \boxed{r = \frac{C}{2\pi}}$$

Example 2. The formula to convert degrees Fahrenheit to degrees Celsius is $\frac{5}{9}(F - 32) = C$ where F = degrees Fahrenheit and C = degrees Celsius. Solve the formula for F .

$$\frac{9}{5} \cdot \frac{5}{9}(F - 32) = C \cdot \frac{9}{5}$$

$$\begin{array}{r} F - 32 = \frac{9}{5}C \\ +32 \quad +32 \end{array}$$

$$\boxed{F = \frac{9}{5}C + 32}$$

Example 3. The formula for computing the volume of a sphere is $V = \frac{4}{3}\pi r^3$ where V = volume and r = length of the radius. Solve the formula for r .

$$V = \frac{4}{3}\pi r^3$$

$$\frac{3}{4}V = \frac{3}{4} \cdot \frac{4}{3}\pi r^3$$

$$\frac{3}{4}V = \pi r^3$$

$$\sqrt[3]{r^3} = \sqrt[3]{\frac{3}{4}\frac{V}{\pi}}$$

$$\boxed{r = \sqrt[3]{\frac{3}{4}\frac{V}{\pi}}}$$

Application 1.

1. Solve $I = prt$ for r .

$$\frac{I}{pt} = \frac{prt}{pt}$$

$$r = \frac{I}{pt}$$

2. Solve $S = 180(n - 2)$ for n .

$$\frac{S}{180} = \frac{180}{180}(n-2)$$

$$\frac{S}{180} = n - 2$$

$$\frac{S}{180} + 2 = n$$

3. Solve $S = 4\pi r^2$ for r .

$$\frac{S}{4\pi} = r^2$$

$$r = \sqrt{\frac{S}{4\pi}}$$

4. Solve $x^2 + y^2 = r^2$ for x .

$$x^2 = r^2 - y^2$$

$$x = \sqrt{r^2 - y^2}$$

Often times in mathematics we are faced with turning real life situations into equations and inequalities.

Example 4: The length of Raelah's rectangular dining room is 12 feet. If the area of the room is at least 96 square feet, what is the smallest width the room can have?



$$A = lw$$

$$\frac{12w}{12} \geq \frac{96}{12}$$

$$w \geq 8 \text{ ft}$$

Example 5. Zac earned \$400 interest in 2016 and \$550 interest in 2017. How much interest must he earn in 2018 so that his average earnings over the three year period is more than \$600?

$$\frac{400 + 550 + x}{3} \geq 600$$

$$950 + x > 1800$$

$$x > 850$$

AT LEAST \$850

Application 2.

5. Mary has a job at a baseball park selling bags of peanuts. She gets paid \$12 per game plus \$1.75 for each bag of peanuts they sell.

a. Create an equation that gives the amount of money, a , Mary will receive if she sells n bags of peanuts.

$$A = 12 + 1.75b$$

$$\begin{array}{r} 53 \\ 1.75 \\ \times 70 \\ \hline 000 \\ 12250 \end{array}$$

b. How much money will she receive for 70 bags?

$$12 + 1.75(70) = 12 + 122.50 \\ = \$134.50$$

c. How many bags does she need to sell to earn \$54?

$$54 = 12 + 1.75b$$

$$42 = 1.75b$$

$$b = 24 \text{ BAGS}$$

d. How many bags does she need to sell to earn at least \$68?

$$12 + 1.75b \geq 68$$

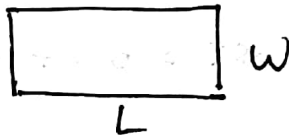
$$b \geq 32$$

$$\text{AT LEAST } 32 \text{ BAGS}$$

$$1.75b \geq 56$$

6. A rectangular dog pen had a perimeter of 100 feet. The length of the rectangle is represented by L and the width is represented by W .

a. Draw and label a diagram to represent the dog pen.



b. Write a formula for the perimeter, P , of the dog pen.

$$P = L + L + W + W$$

$$P = 2L + 2W$$

c. Solve the formula for W .

$$\frac{2W}{2} = \frac{P - 2L}{2}$$

$$W = \frac{P - 2L}{2}$$

d. If the length is 10 feet, what is the width?

$$W(10) = \frac{100 - 2(10)}{2} = \frac{100 - 20}{2} = \frac{80}{2} = 40 \text{ Ft}$$

Practice.

Part 1. Solve each for the indicated variable.

1. $ax - b = c$, for x

$$\frac{ax}{a} = \frac{c+b}{a}$$

$$x = \frac{c+b}{a}$$

2. $15x + 1 = y$, for x

$$\frac{15x}{15} = \frac{y-1}{15}$$

$$x = \frac{y-1}{15}$$

3. $(x + f) + 2 = j$, for x

$$x + f = j - 2$$

$$x = j - 2 - f$$

4. $xy + w = 9$, for y

$$\frac{xy}{x} = \frac{9-w}{x}$$

$$y = \frac{9-w}{x}$$

5. $x(4 - k) = p$, for k

$$4 - k = \frac{p}{x}$$

$$-k = \frac{p}{x} - 4$$

$$k = -\frac{p}{x} + 4$$

6. $7x + 3y = m$, for y

$$\frac{3y}{3} = \frac{m-7x}{3}$$

$$y = \frac{m-7x}{3}$$

7. $4(r+3) = t$, for r

$$\frac{r+3}{4} = \frac{t}{4}$$

$$r+3 = \frac{t}{4}$$

$$r = \frac{t}{4} - 3$$

8. $2x + b = w$, for x

$$2x = w - b$$

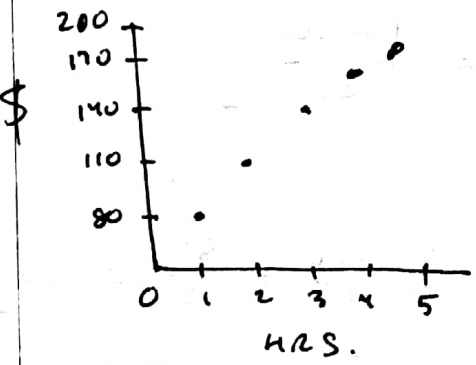
$$x = \frac{w-b}{2}$$

9. $x^3 + 9 = y$, for x

$$x^3 = y - 9$$

$$x = \sqrt[3]{y-9}$$

Part 2. "Rule of 4". Complete the following chart.

<p>Verbal:</p> <p>A car repair will cost a flat fee of \$50 and an additional \$30 per hour for labor costs. We know that the repair will take between <u>one and five</u> hours to complete.</p>	<p>Symbolic:</p> $\text{Cost} = 50 + 30H$												
<p>Graphical:</p> 	<p>Numerical:</p> <table border="1" data-bbox="821 1624 1412 1859"> <thead> <tr> <th>Hour</th> <th>Cost</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>80</td> </tr> <tr> <td>2</td> <td>110</td> </tr> <tr> <td>3</td> <td>140</td> </tr> <tr> <td>4</td> <td>170</td> </tr> <tr> <td>5</td> <td>200</td> </tr> </tbody> </table>	Hour	Cost	1	80	2	110	3	140	4	170	5	200
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