MA4O F2017

Section 1.3: Solving Inequalities

NC.M1.A-REI.3: Solve linear equations and inequalities. Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Review. Solve each equation. Show all steps.

1. $4 = 3a - 14$

$+14 \quad +14$

$\frac{18}{3} = \frac{3a}{3}$

$a = 6$

2. $\frac{a}{5} - 2 = 9$

$5\left(\frac{a}{5} - 2 = 9\right)$

$a - 10 = 45$

$+10 \quad +10$

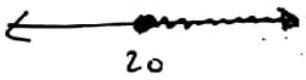
$a = 55$

An open sentence that contains <, >, ≤, or ≥ is called an inequality. The same procedures for solving equations can be used for solving inequalities with some modifications.

Example 1. Solve and graph $x - 12 \geq 8$

$+12 \quad +12$

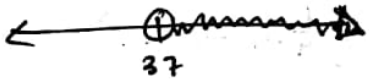
$x \geq 20$



Example 2. Solve and graph $m + 19 > 56$

$-19 \quad -19$

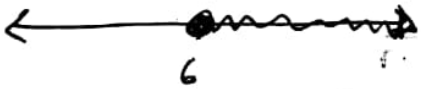
$m > 37$



Example 3. Solve and graph $3a + 6 \leq 4a$

$-3a \quad -3a$

$6 \leq a$

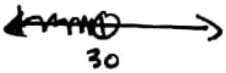


Application 1. Solve and graph each inequality.

1. $22 > m - 8$

$+8 \quad +8$

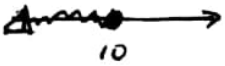
$30 > m$



2. $p + 8 \leq 18$

$-8 \quad -8$

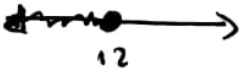
$p \leq 10$



3. $5h \leq 12 + 4h$

$-4h \quad -4h$

$h \leq 12$



When multiplying or dividing both sides of an inequality, special procedures must be followed.

When multiplying or dividing both sides of an inequality by a ***positive*** number, THE INEQUALITY SYMBOL STAYS THE SAME.

When multiplying or dividing both sides of an inequality by a ***negative*** number, THE INEQUALITY SYMBOL IS REVERSED.

Example 4: Solve $\frac{1}{8}n < 84$

$8\left(\frac{1}{8}n < 84\right)$

$n < 672$

Example 5. Solve $-\frac{3}{7}r < 21$

$-\frac{7}{3}\left(-\frac{3}{7}r\right) < 21\left(-\frac{7}{3}\right)$

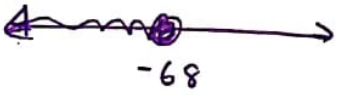
$r > -49$

Application 2. Solve each of the following. Show all steps. Graph each solution.

4. $\frac{1}{4}m \leq -17$

$4\left(\frac{1}{4}m \leq -17\right)$

$m \leq -68$



5. $-10 \leq \frac{x}{-2}$

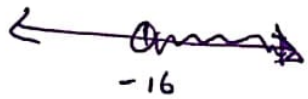
$-2(-10) \leq \frac{x}{-2}(-2)$

$20 \geq x$

6. $32 > -2y$

$\frac{32}{-2} > \frac{-2y}{-2}$

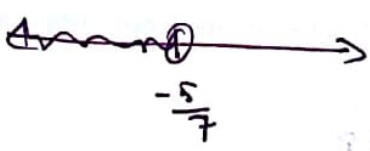
$-16 < y$



7. $-7f > 5$

$\frac{-7f}{-7} > \frac{5}{-7}$

$f < -\frac{5}{7}$



Multi-step inequalities can be solved by the same methods used to solve a multi-step equation while remembering to use the procedures for multiplying and dividing by negative numbers.

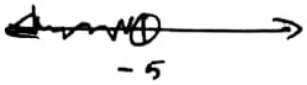
Example 6: Solve and graph $-11y - 13 > 42$

$+13 \quad +13$

$-11y > 55$

$\frac{-11y}{-11} > \frac{55}{-11}$

$y < -5$



Example 7: Solve and graph $4(3t - 5) + 7 \geq 8t + 3$

$12t - 20 + 7 \geq 8t + 3$

$12t - 13 \geq 8t + 3$

$-8t \quad -8t$

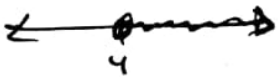
$4t - 13 \geq 3$

$+13 \quad +13$

$4t \geq 16$

$\frac{4t}{4} \geq \frac{16}{4}$

$t \geq 4$



Application 3. Solve and graph each inequality. Show all steps.

8. $9t - 5(t - 5) \leq 4(t - 3)$

$9t - 5t + 25 \leq 4t - 12$

$4t + 25 \leq 4t - 12$

$25 \leq -12$

NO SOL'N

9. $3(4m + 6) \leq 42 + 6(2m - 4)$

$12m + 18 \leq 42 + 12m - 24$

~~NO SOL'N~~

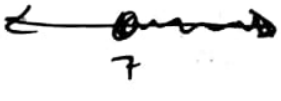
∞ SOL'NS.

10. $6h - 10 \geq 32$

$+10 \quad +10$

$\frac{6h}{6} \geq \frac{42}{6}$

$h \geq 7$



11. $4m - 17 < 6m + 25$

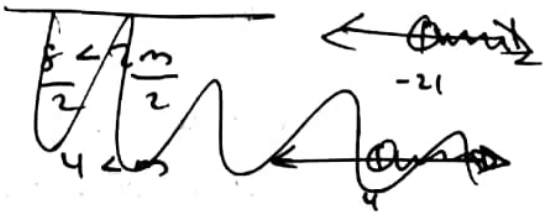
$-4m \quad -4m$

$-17 < 2m + 25$

$+25 \quad +25$

$\frac{-42}{2} < \frac{2m}{2}$

$-21 < m$



18 / 20.

Often times in mathematics inequalities show up in practical applications. Below is a table of examples.

Important Words	Sample Sentence	Equivalent Form	Translation
Is at least	Bill is at least 21 years old.	Bill's age is greater than or equal to 21	$b \geq 21$
Is at most	At most 5 students dropped the course	5 or fewer students dropped the course	$n \leq 5$
Cannot exceed	Earnings cannot exceed $1200	Earnings must be less than or equal to $1200	$r \leq 1200$
Must exceed	The speed must exceed 15 mph	The speed is greater than 15 mph	$s > 15$
Is less than	Spot's weight is less than 50 lb.	---------	$w < 50$
Is more than; is greater than	Boston is more than 200 miles away	---------	$d > 200$
Is between	The film was between 90 and 100 minutes long	---------	$90 < t < 100$

Example 8. Translate each statement into an inequality.

a. The square of m is greater than zero. $m^2 > 0$

b. The reciprocal of t is less than ten. $\frac{1}{t} < 10$

c. The product of r and 5 is at most negative 6. $5r \leq -6$

Example 9. Sarah made an 85 and an 89 on her first two math tests. What must she make on her third test to have at least a 90 average?

$$\frac{85+89+x}{3} \geq 90$$

$$85+89+x \geq 270$$

$$174+x \geq 270$$

$$-174 \quad -174$$

$$x \geq 96$$

Application 4. Solve the following word problems by: (1) clearly identifying your variables, (2) setting up an inequality; (3) solving the inequality (showing all steps); and (4) stating your answer in context of the problem.

12. Robert makes $3.50 per hour working at a convenience store. If he gets a bonus of $25 this week, how many hours must he work to make at least $165?

h = HOURS

$$25 + 3.5h \geq 165$$

$$-25 \quad -25$$

$$3.5h \geq 140$$

$$\frac{3.5h}{3.5} \geq \frac{140}{3.5}$$

$$h \geq 40$$

MUST WORK AT LEAST 40 HRS.

13. The temperatures for the last two days were 28 degrees and 15 degrees. What must the low temperature for the next day be in order for the mean temperature for the 3-day period to be less than 19 degrees?

X = TEMP FOR 3RD DAY

$$\frac{28+15+X}{3} < 19$$

$$28+15+X < 57$$

$$43+X < 57$$

$$-43 \quad -43$$

$$X < 14$$

THE TEMP ON 3RD DAY MUST BE LESS THAN 14^0.

14. The length of a rectangle is one inch less than twice the width. What is the maximum width when the perimeter of the rectangle is no more than 70 inches?

$P = w + w + 2w - 1 + 2w - 1$

$P = 6w - 2$

$6w - 2 \leq 70$

$6w \leq 72$

$w \leq 12$

WIDTH MUST BE $\leq$ 12 IN.

15. If a bracelet costs \$4.75, how many bracelets can Paityn buy for herself and her friends if she wants to spend no more than \$22?

n = # OF BRACELETS.

$$\frac{4.75n}{4.75} \leq \frac{22}{4.75}$$

$$n \leq 4.6$$

4 BRACELETS

Practice. 1st HW →.

1: Solve and graph $\frac{2}{3}x + 9 < 8(\frac{1}{3}x - 2)$

$2x + 27 < 8x - 48$

$-2x \quad -2x$

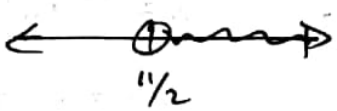
$27 < 6x - 48$

$+16 \quad +16$

$33 < 6x$

$\frac{33}{6} < x$

$\frac{11}{2} < x$



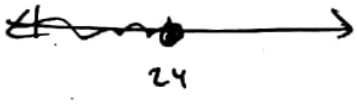
2. Solve and graph $-5 - \frac{t}{6} \geq -9$

$+5 \quad +5$

$-\frac{t}{6} \geq -4$

$-6(-\frac{t}{6}) \geq -4(-6)$

$t \leq 24$



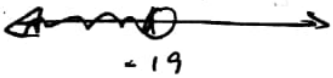
3. Solve and graph $\frac{w+3}{2} < -8$

$2(\frac{w+3}{2}) < -8(2)$

$w + 3 < -16$

$-3 \quad -3$

$w < -19$



4. Solve and graph $3r + 2(4r + 2) \leq 2(6r + 1)$

$3r + 8r + 4 \leq 12r + 2$

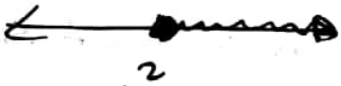
$11r + 4 \leq 12r + 2$

$-11r \quad -11r$

$4 \leq r + 2$

$-2 \quad -2$

$2 \leq r$

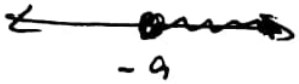


5. Solve and graph $5n - 3(n - 6) \geq 0$

$5n - 3n + 18 \geq 0$

$2n \geq -18$

$n \geq -9$



For each of the following: (1) define a variable; (2) write an inequality; (3) solve. Be sure to give your answer in the context of the problem.

6. A number is less than one fourth the sum of three times the number and four.

$x = \#$

$x < \frac{1}{4}(3x + 4)$

$4x < 3x + 4$

$x < 4$

7. Two times the sum of a number and four is no more than three times the sum of the number and seven decreased by four.

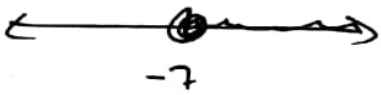
$x = \#$

$2(x+5) \leq 3(x+7) - 4$

$2x + 10 \leq 3x + 21 - 4$

$2x + 10 \leq 3x + 17$

$-7 \leq x$



8. The area of a triangular garden can be no more than 120 square feet. The base of the triangle is 16 feet. Given that the formula for the area of a triangle is $A = \frac{1}{2}bh$, what is the largest the height can be?

$A = \frac{1}{2}bh$

$\frac{1}{2}(16)h \leq 120$

$\frac{8h}{8} \leq \frac{120}{8}$

$h \leq 15$

15 ft

9. Adalae practices the violin at least 12 hours per week. She practices three fourths of an hour each session. IF Adalae has already practiced 3 hours in one week, how many sessions remain to meet or exceed her weekly practice goal?

$n = \#$ OF SESSIONS

$3 + \frac{3}{4}n \geq 12$

$-3 \quad -3$

$\frac{4}{3} \cdot \frac{3}{4}n \geq 9 \cdot \frac{4}{3}$

$n \geq 12$

AT LEAST 12 MORE SESSIONS