

## Section 1.2: Solving Equations and Inequalities

NC.M1.A-REI.3: Solve linear equations and inequalities.

NC.M1.A-REI.1: Understand solving equations as a process of reasoning and explain the reasoning. Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

An **equation** is a mathematical sentence that contains algebraic expressions and symbols and includes an equals sign.

For example  $3x + 7$  is an algebraic expression but  $3x + 7 = 13$  is an equation. Finding a value for a variable that makes the sentence true is called **solving** the equation.

**Example 1:** Solve  $2q + 5 = 13$

$$\begin{array}{r} 2q + 5 = 13 \\ -5 \quad -5 \\ \hline 2q = 8 \\ \frac{2q}{2} = \frac{8}{2} \\ q = 4 \end{array}$$

**Example 2:** Solve  $8m - 7 = 17$

$$\begin{array}{r} 8m - 7 = 17 \\ +7 \quad +7 \\ \hline 8m = 24 \\ \frac{8m}{8} = \frac{24}{8} \\ m = 3 \end{array}$$

**Example 3:** Solve  $\frac{7}{3}y - 8 = 111$

$$\begin{array}{r} 3 \left( \frac{7}{3}y - 8 = 111 \right) \\ 7y - 24 = 333 \\ +24 \quad +24 \\ \hline 7y = 357 \\ \frac{7y}{7} = \frac{357}{7} \\ y = 51 \end{array}$$

**Example 4:** Solve  $5(x + 3) - 3x = 55$

$$\begin{array}{r} 5(x + 3) - 3x = 55 \\ 5x + 15 - 3x = 55 \\ 2x + 15 = 55 \\ -15 \quad -15 \\ \hline 2x = 40 \\ \frac{2x}{2} = \frac{40}{2} \\ x = 20 \end{array}$$

**Application 1.** Solve each of the following equations. Show all steps.

1.  $3x + 2 = 18$

$$\begin{array}{r} 3x + 2 = 18 \\ -2 \quad -2 \\ \hline 3x = 16 \\ \frac{3x}{3} = \frac{16}{3} \\ x = \frac{16}{3} \end{array}$$

2.  $\frac{15}{y} = 3$

$$\begin{array}{r} y \cdot \frac{15}{y} = 3y \\ 15 = 3y \\ \frac{15}{3} = \frac{3y}{3} \\ y = 5 \end{array}$$

3.  $4(y + 1) = 40$

$$\begin{array}{r} 4y + 4 = 40 \\ -4 \quad -4 \\ \hline 4y = 36 \\ \frac{4y}{4} = \frac{36}{4} \\ y = 9 \end{array}$$

4.  $35 = \frac{g-8}{2}$

$$\begin{array}{r} 2(35 = \frac{g-8}{2}) \\ 70 = g - 8 \\ +8 \quad +8 \\ \hline 78 = g \end{array}$$

Example 5: Solve  $4k - 3 = 2k + 5$

$$\begin{array}{r} +3 \quad +3 \\ \hline 4k = 2k + 8 \\ -2k \quad -2k \\ \hline 2k = 8 \\ \frac{2k}{2} = \frac{8}{2} \\ k = 4 \end{array}$$

Example 6: Solve  $2(u + 2) = 3(2u - 7)$

$$\begin{array}{r} 2(u+2) = 3(2u-7) \\ 2u + 4 = 6u - 21 \\ +21 \quad +21 \\ \hline 2u + 25 = 6u \\ -2u \quad -2u \\ \hline 25 = 4u \\ \frac{25}{4} = \frac{4u}{4} \quad u = \frac{25}{4} \end{array}$$

Application 2. Solve each of the following. Show all steps.

5.  $2m + 12 = 3m - 31$

$$\begin{array}{r} -2m \quad -2m \\ \hline 12 = m - 31 \\ +31 \quad +31 \\ \hline m = 43 \end{array}$$

6.  $2h - 8 = h + 17$

$$\begin{array}{r} -h \quad -h \\ \hline h - 8 = 17 \\ +8 \quad +8 \\ \hline h = 25 \end{array}$$

7.  $-7(2b - 4) = 5(-2b + 6)$

$$\begin{array}{r} -14b + 28 = -10b + 30 \\ +10b \quad +10b \\ \hline -4b + 28 = 30 \\ -28 \quad -28 \\ \hline -4b = 2 \\ \frac{-4b}{-4} = \frac{2}{-4} \quad b = -\frac{1}{2} \end{array}$$

8.  $\frac{1}{2} - \frac{5}{8}x = \frac{7}{8}x + \frac{7}{2}$

$$\begin{array}{r} 8\left(\frac{1}{2} - \frac{5}{8}x\right) = 8\left(\frac{7}{8}x + \frac{7}{2}\right) \\ 4 - 5x = 7x + 28 \\ +5x \quad +5x \\ \hline 4 = 12x + 28 \\ -28 \quad -28 \\ \hline -24 = 12x \\ \frac{-24}{12} = \frac{12x}{12} \quad x = -2 \end{array}$$

In middle school you were introduced to solving equations. In high school, not only are we expected to be able to solve them, but we must also justify our methods.

Example 5: The equation  $5(x + 3) - 3x = 55$  is solved below. Justify each step.

$5(x + 3) - 3x = 55$       Given

$5x + 15 - 3x = 55$       Dist. P.

$2x + 15 = 55$       Combine Like Terms.

$$2x + 15 - 15 = 55 - 15 \quad \text{SUBTRACTION P.}$$

$$2x = 40 \quad \text{COMBINE LIKE TERMS.}$$

$$\frac{2x}{2} = \frac{40}{2} \quad \text{DIVISION P.}$$

$$x = 20$$

**Application 3.** Solve each given equation and justify every step.

$$9. 4(y + 1) = 40$$

$$\begin{array}{r} 4y + 4 = 40 \\ -4 \quad -4 \\ \hline 4y = 36 \end{array} \quad \text{DISTR. P.}$$

$$\frac{4y}{4} = \frac{36}{4} \quad \text{SUBST. P.}$$

$$\frac{4y}{4} = \frac{36}{4} \quad \text{DIV. P.}$$

$$y = 9$$

$$10. \frac{w-4}{5} = -3$$

$$5 \left( \frac{w-4}{5} = -3 \right) \quad \text{MULT. P.}$$

$$\begin{array}{r} w-4 = -15 \\ +4 \quad +4 \\ \hline w = -11 \end{array} \quad \text{ADD. P.}$$

$$w = -11$$

$$11. 11x - 4 = 29$$

$$\begin{array}{r} 11x - 4 = 29 \\ +4 \quad +4 \\ \hline 11x = 33 \end{array} \quad \text{ADD. P.}$$

$$\frac{11x}{11} = \frac{33}{11} \quad \text{DIV. P.}$$

$$x = 3$$

$$12. 2 + 5k = 3k - 6$$

$$\begin{array}{r} 2 + 5k = 3k - 6 \\ -3k \quad -3k \\ \hline 2 + 2k = -6 \end{array} \quad \text{SUBST. P.}$$

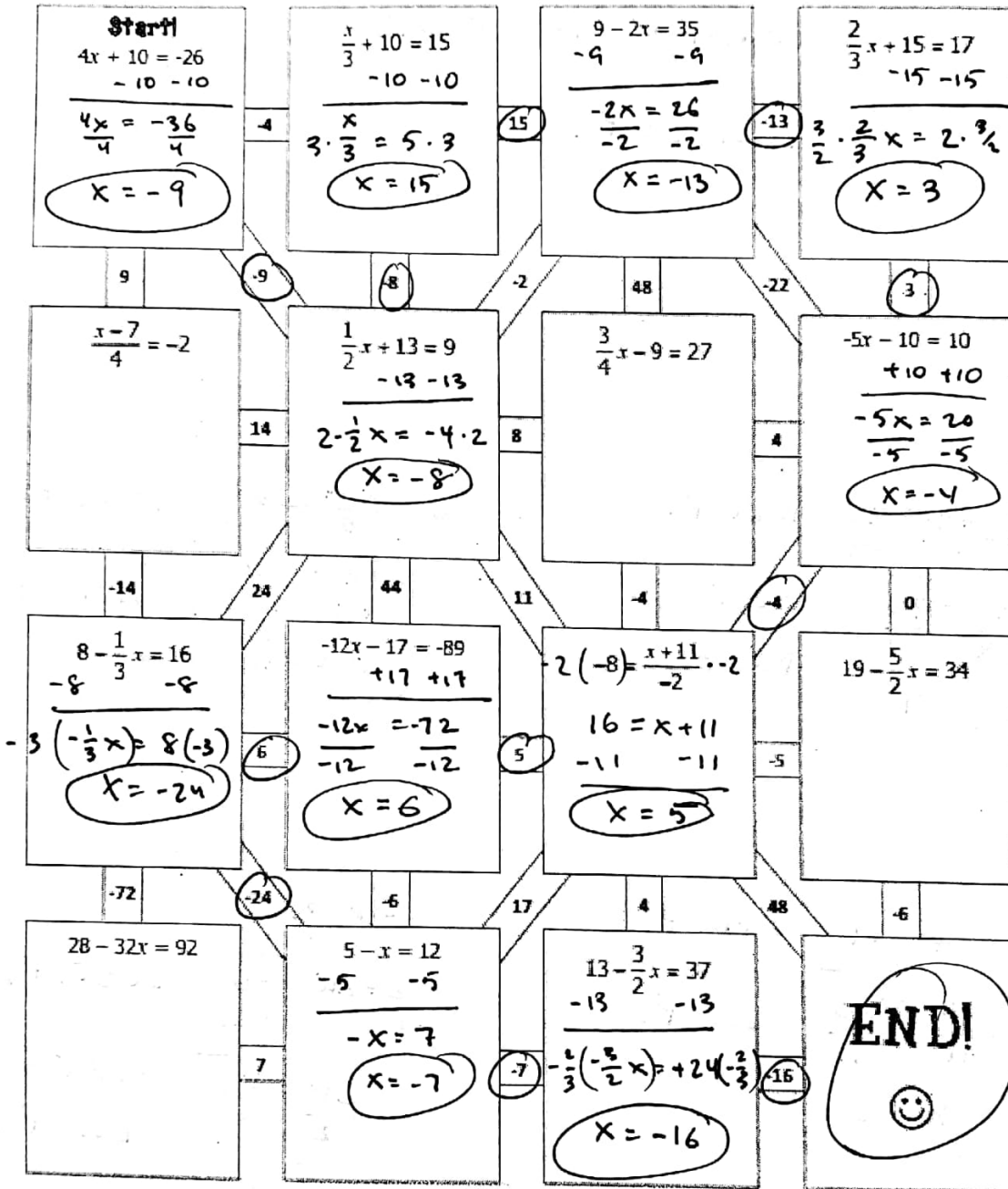
$$\begin{array}{r} 2 + 2k = -6 \\ -2 \quad -2 \\ \hline 2k = -8 \end{array} \quad \text{SUBST. P.}$$

$$\frac{2k}{2} = \frac{-8}{2} \quad \text{DIV. P.}$$

$$k = -4$$

Activity.

Directions: Use your solutions to navigate through the puzzle. SHOW ALL STEPS!!!!



Practice. Solve each equation. Show all steps. Circle final answer.

<p>1. <math>3(2x + 1) - 7 = 50</math>  <math>6x + 3 - 7 = 50</math>  <math>6x - 4 = 50</math>  <math>6x = 54</math>  <math>x = 9</math></p>	<p>6. <math>4(x - 1) = 3(2x - 6) + 4</math>  <math>4x - 4 = 6x - 18 + 4</math>  <math>4x - 4 = 6x - 14</math>  <math>4x + 10 = 6x</math>  <math>2x = 10</math> <math>x = 5</math></p>
<p>2. <math>8 - (2x - 6) = 22</math>  <math>8 - 2x + 6 = 22</math>  <math>14 - 2x = 22</math>  <math>-2x = 8</math>  <math>x = -4</math></p>	<p>7. <math>\frac{2}{3}x + 9 = 8(\frac{1}{2}x - 2)</math>  <math>6(\frac{2}{3}x + 9) = 8(\frac{1}{2}x - 2)</math>  <math>4x + 54 =</math>  <math>\frac{2}{3}x + 9 = 4x - 16</math>  <math>2x + 27 = 12x - 48</math> <math>10x = 75</math>  <math>x = 7.5</math></p>
<p>3. <math>3(x - 5) = -2(4 - 5x)</math>  <math>3x - 15 = -8 + 10x</math>  <math>-3x \quad -3x</math>  <math>-15 = -8 - 3x</math>  <math>+8 \quad +8</math>  <math>-7 = -3x</math>  <math>x = \frac{7}{3}</math></p>	<p>8. <math>\frac{7}{3}y - 8 = 111</math>  <math>7y - 24 = 333</math>  <math>+24 \quad +24</math>  <math>7y = 357</math>  <math>y = 71</math></p>
<p>4. <math>4(x + 1) - 3(2x + 6) = -11</math>  <math>4x + 4 - 6x - 18 = -11</math>  <math>-2x - 14 = -11</math>  <math>+14 \quad +14</math>  <math>-2x = 3</math>  <math>x = -\frac{3}{2}</math></p>	<p>9. <math>\frac{1}{5}(10 - 20x) = -14</math>  <math>2 - 4x = -14</math>  <math>-2 \quad -2</math>  <math>-4x = -16</math>  <math>-4 \quad -4</math>  <math>x = 4</math></p>
<p>5. <math>2(x - 8) + 7 = 5(x + 2) - 3x - 19</math>  <math>2x - 16 + 7 = 5x + 10 - 3x - 19</math>  <math>2x - 9 = 2x - 9</math>  <math>(\infty \text{ SOLUTIONS})</math></p>	<p>10. <math>\frac{5}{2} - x = 3 + \frac{3}{2}x</math>  <math>2(\frac{5}{2} - x) = 3 + \frac{3}{2}x</math>  <math>5 - 2x = 6 + \frac{3}{2}x</math>  <math>+2x \quad +2x</math>  <math>5 = 6 + 5x</math>  <math>-6 \quad -6</math>  <math>-1 = 5x</math>  <math>x = -\frac{1}{5}</math></p>