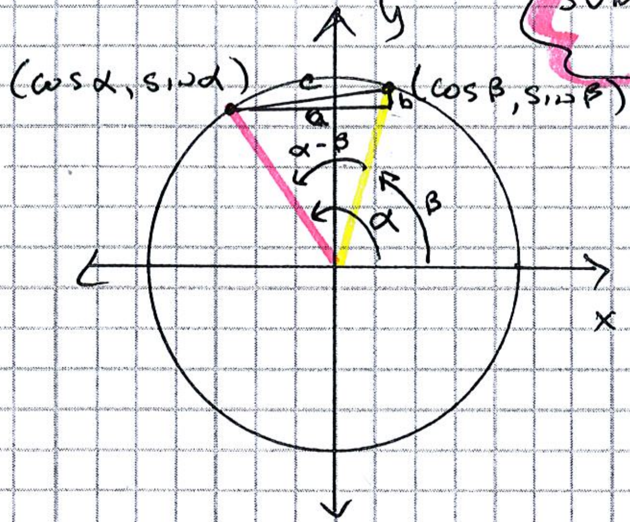
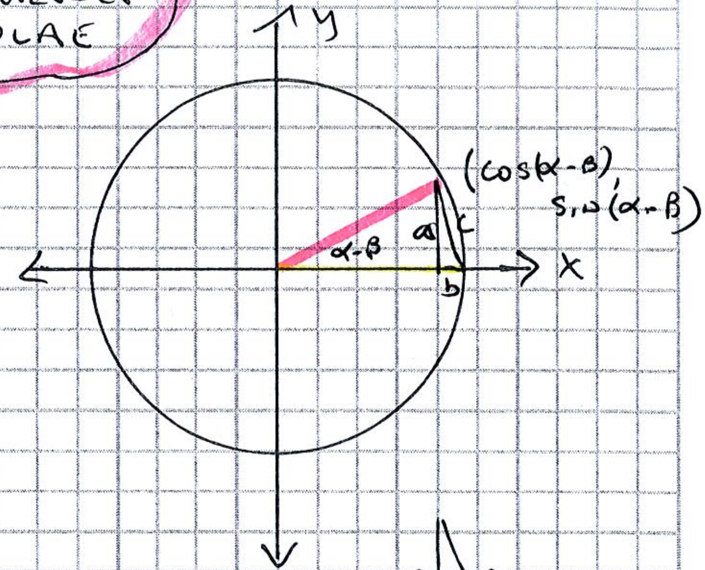
DERIVATION OF SUM/DIFFERENCE FORMULAE





$a^2 + b^2 = c^2$

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SUBSTITUTE! $a = \cos\beta - \cos\alpha$

$b = \sin\beta - \sin\alpha$

$(\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin\alpha)^2 = c^2$

$\underline{\cos^2\beta} - 2\cos\alpha\cos\beta + \underline{\underline{\cos^2\alpha}} + \underline{\sin^2\beta} - 2\sin\alpha\sin\beta + \underline{\underline{\sin^2\alpha}} = c^2$

$\underline{1} + 1 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta = c^2$

$2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = c^2$ ①

SUBSTITUTE! $a = \sin(\alpha - \beta)$

$b = 1 - \cos(\alpha - \beta)$

$\sin^2(\alpha - \beta) + (1 - \cos(\alpha - \beta))^2 = c^2$

$\underline{\sin^2(\alpha - \beta)} + 1 - 2\cos(\alpha - \beta) + \underline{\cos^2(\alpha - \beta)} = c^2$

$1 + 1 - 2\cos(\alpha - \beta) = c^2$

$2 - 2\cos(\alpha - \beta) = c^2$ ②

① = ② $\Rightarrow$ $2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 2 - 2\cos(\alpha - \beta)$

$-2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = -2\cos(\alpha - \beta)$

$\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha - \beta)$

$$\boxed{\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

LET $\beta = -\beta \Rightarrow$

$$\cos(\alpha - (-\beta)) = \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta)$$

$$\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

SINCE $\cos(-\beta) = \cos\beta$
AND $\sin(-\beta) = -\sin\beta$
(EVEN + ODD)

LET $\alpha = \frac{\pi}{2} - \alpha$

$$\cos\left(\frac{\pi}{2} - \alpha + \beta\right) = \cos\left(\frac{\pi}{2} - \alpha\right)\cos\beta - \sin\left(\frac{\pi}{2} - \alpha\right)\sin\beta$$

$$\cos\left(\frac{\pi}{2} - (\alpha - \beta)\right) = \cos\left(\frac{\pi}{2} - \alpha\right)\cos\beta - \sin\left(\frac{\pi}{2} - \alpha\right)\sin\beta$$

$$\boxed{\sin(\alpha - \beta) = \sin(\alpha)\cos\beta - \cos\alpha\sin\beta}$$

(PHASE SHIFT)

LET $\beta = -\beta$.

$$\sin(\alpha - (-\beta)) = \sin\alpha\cos(-\beta) - \cos\alpha(\sin(-\beta))$$

$$\boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

(ODD/EVEN)

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \Rightarrow \tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$= \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$= \frac{\frac{\sin\alpha\cos\beta}{\cos\alpha} + \frac{\sin\beta\cos\alpha}{\cos\alpha}}{\frac{\cos\alpha\cos\beta}{\cos\alpha} - \frac{\sin\alpha\sin\beta}{\cos\alpha}} = \frac{\tan\alpha\cos\beta + \sin\beta}{\cos\beta - \tan\alpha\sin\beta} =$$

$$\frac{\frac{\tan\alpha\cos\beta}{\cos\beta} + \frac{\sin\beta}{\cos\beta}}{\frac{\cos\beta}{\cos\beta} - \frac{\tan\alpha\sin\beta}{\cos\beta}} = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\boxed{\tan\alpha+\beta = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$$

$$\tan\alpha-\beta = \frac{\tan\alpha + \tan(-\beta)}{1 - \tan\alpha\tan(-\beta)} = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\boxed{\tan\alpha-\beta = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$$