Section 9.2 - Tests About a Population Proportion (pp. 554-574)

A significance test uses sample data to measure the strength of evidence against H_0. Here are some principles that apply to most tests:

- The test compares a statistic calculated from sample data with the value of the parameter stated by the null hypothesis.
- Values of the statistic far from the null parameter value in the direction specified by the alternative hypothesis give evidence *against* H_0.
- To assess how far the statistic is from the parameter, *standardize* the statistic.

Test Statistic

A **test statistic** measures how far a sample statistic *diverges* from what we would expect if the null hypothesis H_0 were true, in standardized units. That is,

$$test\ statistic = \frac{statistic - parameter}{standard\ deviation\ of\ statistic}$$

Significance Tests: A Four-Step Process

State: What *hypotheses* do you want to test, and at what *significance level*? Define any *parameters* you use.

P = clearly state the parameter of interest
H = clearly state the null and alternative hypotheses

Plan: Choose the appropriate *method*. Check *assumptions/conditions*.

A = write all assumptions/check conditions
N = name the hypothesis test you will be performing

Do: If the conditions are met, perform the *calculations*.

T = use the formula to find the test statistic
O = obtain the *P*-value

Conclude: *Interpret* the results of your test in the context of the problem.

M = "mantra" conclusion

PHANTOM

When the conditions are met -- Random, Normal, and Independent -- the sampling distribution of $\hat{p}$ is approximately Normal with

mean $\mu_{\hat{p}} = p$ and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

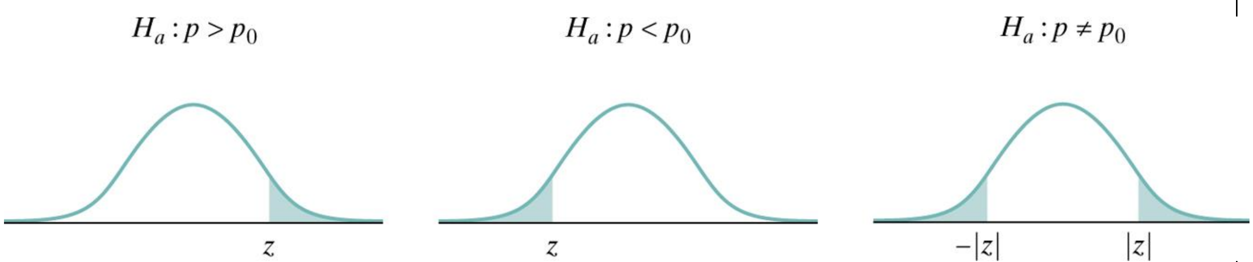
For *confidence intervals*, we substitute $\hat{p}$ for *p* in the standard deviation formula to obtain the standard error. When performing *significance tests*, however, the null hypothesis specifies a value for *p*, which we will call p_0. When we standardize the statistic $\hat{p}$ by substituting its mean and standard deviation we get the test statistic. This leads to the *One-Sample z-Test for a Proportion*.

One-Sample z-Test for a Proportion

Choose an SRS of size *n* from a large population that contains and unknown proportion *p* of successes. To test the hypothesis H_0: $p = p_0$, compute the z-statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

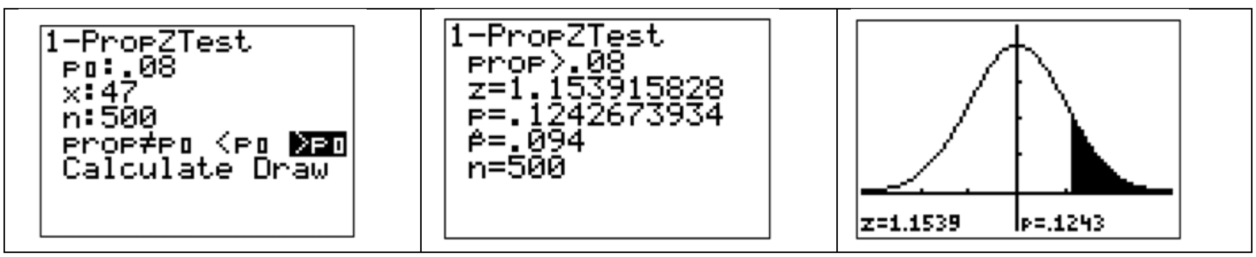
Finding the *P*-value by calculating the probability of getting a z statistic this large or larger in the direction specified by the alternative hypothesis H_A.



Example (One-tailed test) - On shows like *American Idol*, contestants often wonder if there is an advantage to performing last. To investigate this, a random sample of 600 AI fans is selected to view the audition tapes of 12 never-before-seen contestants. For each fan, the order of the 12 videos is randomly determined. Thus, if the order of performance does not matter, we would expect exactly ½ of the fans to prefer the last contestant they viewed. Do these data provide convincing evidence that there is an advantage to going last?

Application - Complete CYU on page 560.

Technology



Example (two-sided test) - According to the Centers for Disease Control and Prevention (CDC) Web site, 50% of high school students have never smoked a cigarette. Joe wants to know whether this result holds true in his large, urban high school. He surveys an SRS of 150 students from his school. He gets responses from all 150 students, and 90 say they have never smoked a cigarette. What should Joe conclude? Give appropriate evidence to support your answer.

Application - Complete CYU on p. 563.

Why Confidence Intervals Give More Information

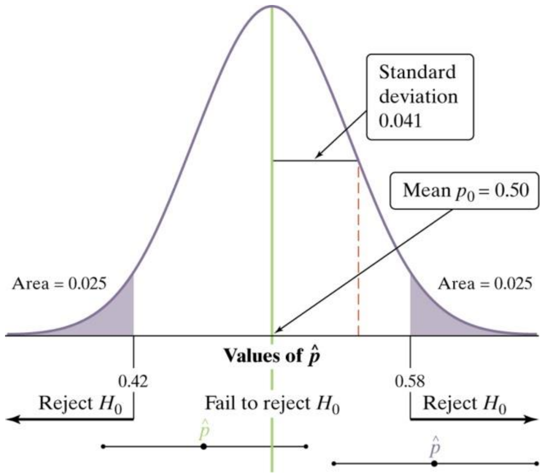
In a hypothesis test, we basically make a decision to reject H_0 or fail to reject H_0. We are still left wondering what the actual population parameter is. A confidence interval might shed light on this.

Example (cont) - Joe found that 90 of an SRS of 150 students said they had smoked a cigarette. We checked the conditions for performing the significance test earlier but we used $\hat{p} = p_0$. For a confidence interval we have to check using $\hat{p} = 0.6$.

Based on the confidence interval, what can we say?

The confidence interval is much more informative in this example than the significance test we performed.

There is a link between confidence intervals and two-sided tests. The 95% confidence interval (0.522, 0.678) gives an approximate range of p_0's that would not be rejected by a two-sided test at the $\alpha = 0.05$ significance level. With proportions, the link is not perfect because the standard error for the confidence interval is based upon the sample proportion $\hat{p}$ while the denominator of the test statistic is based on p_0.



H_0: p = 0.50
H_A: p ≠ 0.50
α = 0.05

Example

Session

Test and Confidence Interval for One Proportion

Test of p = 0.75 vs p not = 0.75

Sample	X	N	Sample p	95.0 % CI	Z-Value	P-Value
1	68	100	0.680000	(0.588572, 0.771428)	-1.62	0.106

HW: Read Sec 9.2; do problems 27-43, 45, 47, 51, 53, 55, 57.