**Section 6.1 Discrete and Continuous Random Variables**

**1. Random Variables.** Consider tossing a fair coin 3 times. The sample space would be:

S = { HHH, HHT, HTH, THH, HTT, THT, TTH, TTT }

Let **X** represent the number of heads obtained. We can depict this situation in a **probability distribution of X**:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Value** | 0 | 1 | 2 | 3 |
| **Probability** |  |  |  |  |

We can use the probability distribution to answer questions about the variable X such as what   
is P(X ≥ 1)?

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| **Definition:** A **random variable** takes numerical values that describe the outcomes of some chance process. The **probability distribution** of a random variable gives its possible values and their probabilities. |

**2. Discrete Random Variables**

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| **Definition**: A **discrete random variable** X takes on a fixed set of possible values with gaps between. The probability distribution of a discrete random variable X lists the values xi and their probabilities pi:  **Value:** x1 x2 x3 . . .  **Probability:** p1 p2 p3 . . .  The probabilities *pi* must satisfy two requirements:  1. Every probability *pi* is a number between 0 and 1.  2. The sum of the probabilities is 1.  To find the probability of any event, add the probabilities *pi* of the particular values of *xi* that make up that event. |

**Example** - In 2010, there were 1319 games played in the National Hockey League’s regular season. Imagine selecting one of these games at random and then randomly selecting one of the two teams that played in the game. Define the random variable X = number of goals scored by a randomly selected team in a randomly selected game. The table below gives the probability distribution of X:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Goals | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Probability | 0.061 | 0.154 | 0.228 | 0.229 | 0.173 | 0.094 | 0.041 | 0.015 | 0.004 | 0.001 |

(a) Show that the probability distribution for X is legitimate.

(b) What is the probability that the number of goals scored by a randomly selected team in a randomly selected game is at least 6?

**Check Your Understanding** - Complete CYU on p. 350.

**3. The Mean (Expected Value) of a Discrete Random Variable**

When analyzing shapes of distributions we used **SOCS**. If we want to know the center of a distribution of a discrete random variable we are going to have to compute the mean. The mean of a discrete random variable X is denoted by **μx** . It is an average of all possible values of the random variable X but we have to take into account how many times we *expect* the values to occur. For this reason the mean is also referred to as the **expected value** of the random variable.

**Example**: Given the probability distribution of the discrete random variable X, find the expected value of X.

|  |  |  |  |
| --- | --- | --- | --- |
| Value | 1 | 2 | 3 |
| Probability | 0.5 | 0.2 | 0.3 |

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| **Definition:** Suppose that X is a discrete random variable whose probability distribution is  **Value:** x1 x2 x3 . . .  **Probability:** p1 p2 p3 . . .  To find the **mean (expected value)** of X, multiply each possible value by its probability then add all the products: |

**Example:** Find the expected value of the random variable X in the NHL example and interpret the value in context.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Goals | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Probability | 0.061 | 0.154 | 0.228 | 0.229 | 0.173 | 0.094 | 0.041 | 0.015 | 0.004 | 0.001 |

**Note**: *A common error on the AP Exam is that students incorrectly believe that the expected value of a random variable must be equal to one of the possible values of the variable. This is not the case.*

**4. The Standard Deviation (and Variance) of a Discrete Random Variable**

In order to describe the *spread* of the distribution of a discrete random variable, we are going to use the standard deviation. In order to find the standard deviation, we first compute the variance and then find its square root. The variance is the *average of the squared deviation of the possible X values from the mean*. Again, however, we must take into account how often we expect the different values of X to occur.

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| **Definition:** Suppose that X is a discrete random variable whose probability distribution is  **Value:** x1 x2 x3 . . .  **Probability:** p1 p2 p3 . . .  and that μx is the mean of X. The **variance of X** is  The **standard deviation** of X, **σx** is the square root of the variance. |

**Example**. Compute and interpret the standard deviation of the random variable X in the NHL example and interpret its meaning in context.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Goals | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Probability | 0.061 | 0.154 | 0.228 | 0.229 | 0.173 | 0.094 | 0.041 | 0.015 | 0.004 | 0.001 |

**Check Your Understanding** - Complete CYU on p. 355.

**5. Continuous Random Variables**

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| **Definition**: A **continuous random variable** X takes all values in an interval of numbers. The probability distribution of X is described by a *density curve*. The probability of any event is the area under the density curve and above the values of X that make up that event. |

The most familiar continuous probability distribution is the (vaunted) **Normal distribution**.

**Example:** The weights of three-year-old females closely follow a *Normal* *distribution* with a mean of μ = 30.7 pounds and a standard deviation of 3.6 pounds. Randomly choose one three-year-old female and call her weight X. Find the probability that the randomly selected female weighs at least 30 pounds.

**State**:

**Plan:**

**Do:**

**Conclude**:

HW: 1, 5, 7, 9, 13, 14, 18, 19, 24, 33\*, 34\*