**Section 5.3 - Conditional Probability & Independence** (pp. 318-333)

**1. Conditional** Probability - Let’s return to the setting of the homeowners example in Section 5.2.

|  |  |  |  |
| --- | --- | --- | --- |
|  | High School Grad | Not a HS Grad | Total |
| Homeowner | 221 | 119 | 340 |
| Not a homeowner | 89 | 71 | 160 |
| Total | 310 | 190 | 500 |

If we know that a person owns a home, what is the probability that the person is a high school graduate?

If we know that a person is a high school graduate, what is the probability that the person owns a home?

These questions involve **conditional probabilities**. The name comes from the fact that we are trying to find the probability that one event will happen under the *condition* that some other event is already known to have occurred. We often use the phrase *“given that”* to signal the condition.

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| **Definition**: The probability that one event happens given that another event is already known to have happened is called a **conditional probability**. Suppose we know that event A has happened. Then the probability that A happens *given that* event B has happened is denoted by P(A | B). |

Using this notation, we can restate the answers to our two previous questions:

* P(HS grad | Homeowner) =
* P(Homeowner | HS grad)=

**2. Calculating Conditional Probabilities**

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| **Conditional Probability Formula**  To find the conditional probability P(B | A), use the formula |

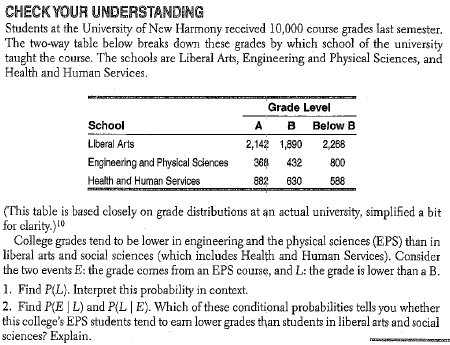
**Example**: Given the table below, what is the probability that a randomly selected household with a landline also has a cell phone?

|  |  |  |  |
| --- | --- | --- | --- |
|  | Cell Phone | No Cell Phone | Total |
| Landline | 0.60 | 0.18 | 0.78 |
| No Landline | 0.20 | 0.02 | 0.22 |
| Total | 0.80 | 0.20 | 1.00 |

Is there a connection between *conditional probability* and the *conditional distribution* from Chapter 1?

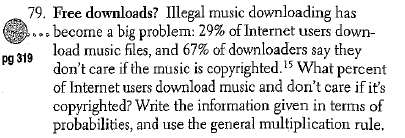
The answer is **yes**. The two segmented bar graphs below display the conditional distributions for the Homeowners example.

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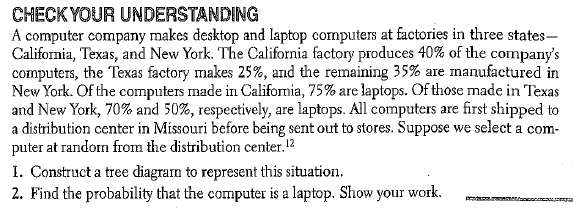
**3. The General Multiplication Rule**

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| **General Multiplication Rule**  The probability that events A and B both occur can be found using the **general multiplication rule**  Where P(B|A) is the conditional probability that event B occurs given that event A has already occurred. |



**4. Tree Diagrams and the General Multiplication Rule**

Shannon hits the snooze bar on her alarm clock on 60% of school days. If she does not hit the snooze bar, there is a 0.90 probability that she makes it to class on time. However, if she hits the snooze bar, there is only 0.70 probability that she makes it to class on time. In a randomly chosen day, what is the probability that Shannon is late to class?



**5. Conditional Probability and Independence**

Suppose you toss a fair coin twice. Define events A: first toss is a head, and B: second toss is a head. P(A) = 0.5 and P(B) = 0.5. What is P(A|B)? It is the conditional probability that the second toss is a head given that the first toss was a head. The coin has no memory, so P(A|B) = 0.5. In this case P(A|B) = P(A).

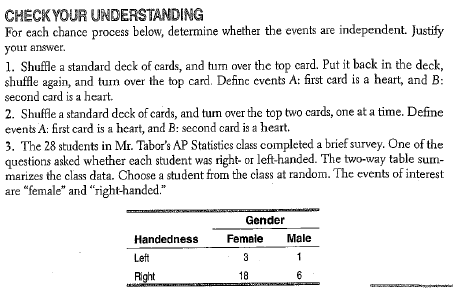
Let’s contrast the coin-toss scenario with our earlier homeowner example. The events of interest were A: is a high school graduate and B: owns a home. We already learned that P(B) = 340/500 = 0.68 and P(B|A) = 221/310 = 0.712. That is, we know that a randomly selected member of the sample has a 0.68 probability of owning a home. However, if we know that the randomly selected member is a high school graduate, the probability of owning a home increases to 0.712.

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| **Definition**. Two events A and B are **independent** if the occurrence of one event has no effect on the chance that the other event will happen. In other words, events A and B are independent if |

**Example** - Is there a relationship between gender and having allergies? To find out, we used the random the CensusAtSchool web site to randomly select 40 U.S. high school students who completed a survey. The two-way table shows the gender of each student and whether the student has allergies.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Female | Male | Total |
| Allergies | 10 | 8 | 18 |
| No Allergies | 13 | 9 | 22 |
| Total | 23 | 17 | 40 |

Are the events “female” and “allergies” independent?



**5. Independence: A Special Multiplication Rule** - What happens to the general multiplication rule when events A and B are independent?

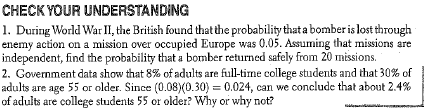
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| **Multiplication Rule for Independent Events**  If A and B are independent events, then the probability that A and B both occur is |

**Example**: In baseball, a perfect game is when a pitcher does not allow any hitters to reach base in all nine innings. Historically, pitchers throw a perfect inning - an inning where no hitters reach base - about 40% of the time. So, to throw a perfect game, a pitcher needs to have nine perfect innings in a row.

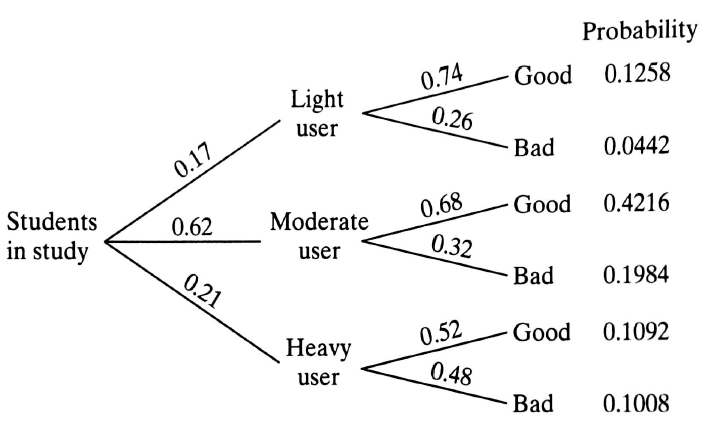
What is the probability that a pitcher throws nine perfect innings in a row, assuming the pitcher’s performance in an inning is independent of his performance in the other innings.

**Example**: The First Trimester Screening is a noninvasive test given during the first trimester of pregnancy to determine if there are specific chromosomal abnormalities in the fetus. According to the *New England Journal of Medicine* in November 2005, approximately 5% of normal pregnancies will receive a false positive result.

Among 100 women with normal pregnancies, what is the probability that there will be *at least* 1 false positive?



**Example:** Given the diagram below, what percent of youth with good grades are heavy users of media?



**Example:** Many employers require prospective employees to take a drug test. A positive result indicates that the prospective employee uses illegal drugs. However, not all people who test positive actually use drugs. Suppose that 4% of the prospective employees use drugs, the false positive rate is 5% and the false negative rate is 10%.

What percent of people who test positive actually use drugs?

HW: 63, 65, 67, 69, 75, 79, 81, 97-99