ACTIVITY 4.1

1. Complete each quotient or reciprocal identity.
   a. $\cos x =$
   b. $\tan x =$
   c. $\csc x =$
2. Complete each Pythagorean identity.
   a. $\sin^2\theta + \cos^2\theta =$
   b. $\tan^2 x + 1 =$
3. Simplify each expression.
   a. $\sin x \tan x \csc^2 x$
   b. $(\sec x + 1)(\sec x - 1)$
   c. $\cot\theta + \tan\theta$

Verify each identity.

4. $\cot x \csc x \tan x = \frac{1}{\sin x}$
5. $\frac{\sin^2\theta}{1-\cos\theta} = 1 + \cos\theta$
6. $(\tan\theta + 1)^2 = 2\tan\theta + \sec^2\theta$
7. $\frac{1+\tan x}{1+\cot x} = \tan x$

ACTIVITY 4.2

8. Complete each statement.
   a. If $\cos(\theta) = b$, then $\cos(\theta + 180°) =$ _____.
   b. If $\sin(-x) = \frac{3}{5}$, then $\sin(x) =$ _____.
   c. If $\cos(x) = 0.75$, then $\sin\left(\frac{\pi}{2} - x\right) =$ _____.
   d. If $\sin(52°) = y$, then $\cos(38°) =$ _____ and $\sin(232°) =$ _____.
   e. If $\cos\left(\frac{\pi}{8}\right) = y$, then $\sin\left(\frac{3\pi}{8}\right) =$ _____ and $\cos\left(\frac{7\pi}{8}\right) =$ _____.
9. Which of the following statements below are identities? Explain your reasoning.
   a. $\tan(\theta) = \cot(90° - \theta)$
   b. $\sin(x) = -\sin(-x)$
   c. $\cos(x) = \cos(\pi + x)$
   d. $\sin(x) = -\cos\left(\frac{\pi}{2} + x\right)$
10. Solve each equation over the given interval. You do not need a calculator.
    a. $4\cos^2 x - 1 = 0, [0, 2\pi)$
    b. $2\sin\theta\cos\theta = \cos\theta, [0°, 360°)$
    c. $\cos^2 x - 2\sin x - 1 = 0, [0, 2\pi)$
11. Solve each equation over the interval $(-\infty, \infty)$. You do not need a calculator.
    a. $\sec x - 2\cos x = 0$
    b. $1 - \sin\theta = 2\cos^2\theta$
12. Solve each equation over the given interval. You may use a calculator.
    a. $\tan^2\theta - 3\tan\theta = 4, [0°, 360°)$
    b. $\sin x = 2\cos x, [0, 2\pi)$

ACTIVITY 4.3

13. Verify each identity.
    a. $\sin(x + 90°) = \cos x$
    b. $\tan(x + 45°) = \frac{\tan x + 1}{1 - \tan x}$
14. Find the exact value.
    a. $\cos 15°$
    b. $\sin 75°$
    c. $\cos\left(\frac{-13\pi}{12}\right)$
15. Given $\sin a = \frac{-5}{13}$, $\cos b = \frac{4}{5}$, a terminates in Quadrant IV, and b terminates in Quadrant I. Find the exact value of each ratio.
    a. $\sin(a + b)$
    b. $\cos(a - b)$
    c. $\tan(a + b)$
16. Given acute angle θ with $\sin\theta = \frac{1}{3}$. Find the exact value of each ratio.
    a. $\sin 2\theta$
    b. $\cos 2\theta$
17. Find the requested ratio
    a. $\tan(22.5°)$
    b. $\cos\left(\frac{11\pi}{12}\right)$
    c. $\sin\left(\frac{\theta}{2}\right)$ if $\cos\theta = \frac{-3}{5}$ and θ terminates in Quadrant II.

© 2010 College Board. All rights reserved.

7. $\frac{1+\tan x}{1+\cot x} = \frac{1+\frac{\sin x}{\cos x}}{1+\frac{\cos x}{\sin x}} = \frac{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}} =$

$\frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\sin x+\cos x}{\sin x}} = \frac{\cos x+\sin x}{\cos x}\cdot\frac{\sin x}{\sin x+\cos x} = \frac{\sin x}{\cos x} = \tan x$

UNIT 4 PRACTICE

Activity 4.1

1a. $\cos\theta = \frac{1}{\sec\theta}$
b. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
c. $\csc\theta = \frac{1}{\sin\theta}$

2a. $\sin^2\theta + \cos^2\theta = 1$
b. $\tan^2 x + 1 = \sec^2 x$

3a. $\sec x$
b. $\tan^2 x$
c. $\sec\theta\csc\theta$

4. $\cot x \csc x \tan x =$
$\frac{\cancel{\cos x}}{\cancel{\sin x}}\frac{1}{\sin x}\frac{\cancel{\sin x}}{\cancel{\cos x}} =$
$\frac{1}{\sin x} = \frac{1}{\sin x}$

5. $\frac{\sin^2\theta}{1-\cos\theta} =$
$\frac{1-\cos^2\theta}{1-\cos\theta} =$
$\frac{\cancel{(1-\cos\theta)}(1+\cos\theta)}{\cancel{1-\cos\theta}} =$
$1 + \cos\theta = 1 + \cos\theta$

6. $(\tan\theta + 1)^2 =$
$\tan^2\theta + 2\tan\theta + 1 =$
$2\tan\theta + \sec^2\theta = 2\tan\theta + \sec^2\theta$

7. (See below)

Activity 4.2

8a. $-b$ b. $\frac{-3}{5}$ c. 0.75
d. $y, -y$ e. $y, -y$

9. Answers may vary. Sample explanations:
a. Yes. Cofunction identity
b. Yes. Sine is an odd function
c. No. $\cos(\pi + x) = -\cos x$, because $\pi + x$ terminates in quadrant 3.
d. Yes. The graphs of each side of the equation are identical.

10a. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
b. 90°, 270°, 30°, 150°
c. $0, \pi$

11a. $45° + 90°k$, where k is an integer.
b. $210° + 360°k$, $330° + 360°k$, $90° + 360°k$, where k is an integer.

12a. 75.964°, 135°, 255.964°, 315°
b. 1.107, 4.249

Scanned by CamScanner

Activity 4.3

13a. See bottom right.

b. $\tan(x + 45°) = \dfrac{\tan x + \tan 45°}{1 - \tan x \tan 45°} = \dfrac{\tan x + 1}{1 - \tan x}$

14a. $\dfrac{\sqrt{6} + \sqrt{2}}{4}$

b. $\dfrac{\sqrt{2} + \sqrt{6}}{4}$

c. $\dfrac{-\sqrt{6} - \sqrt{2}}{4}$

15a. $\dfrac{16}{65}$

b. $\dfrac{33}{65}$

c. $\dfrac{16}{63}$

16a. $\dfrac{4\sqrt{2}}{9}$

b. $\dfrac{7}{9}$

17a. $\sqrt{\dfrac{2 - \sqrt{2}}{2 + \sqrt{2}}}$

b. $-\dfrac{\sqrt{2 + \sqrt{3}}}{2}$

c. $\dfrac{2\sqrt{5}}{5}$

18a.–c. See bottom right.

19a. $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

b. $\dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8}, \dfrac{13\pi}{8}$

c. $0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

Activity 4.4

20a. 430.182 **b.** 31.305

21. 25.456

22. 55.801

23. $25(\tan 53°) + 3 = 36.176$ ft

24. The distance is $150(\tan 68°) \approx 371.263$ ft

25. Yes, the ramp will pass inspection because the angle of elevation is $\tan^{-1}\left(\dfrac{30 \text{ in.}}{360 \text{ in.}}\right) \approx 4.763° < 4.8°$.

Activity 4.5

26a. 7 m

b. 16.52 m

c. 17 m

d. $7 \le d \le 17$

27. $d = \sqrt{(5\sin\theta)^2 + (12 - 5\cos\theta)^2}$

28. 54.9°, 305.10°

18. Verify each identity

a. $\tan(x + 360°) = \tan x$

b. $\sin(x + \pi) = -\sin x$

c. $2\csc 2x = \csc x \sec x$

19. Solve each equation on the interval $[0, 2\pi)$.

a. $\cos 2x = 2\sin^2 x$

b. $\dfrac{2\tan x}{1 - \tan^2 x} = 1$

c. $\sec 3x - 1 = 0$

ACTIVITY 4.4

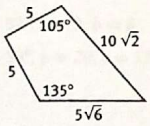
20. Find the area of each triangle.

a. $A = 35°, b = 30, c = 50$

b. $a = 7, b = 12, c = 9$

21. Find the area of a regular octagon inscribed in a circle whose radius is 3.

22. Find the area of the quadrilateral shown below. (Hint: Divide it into two triangles).



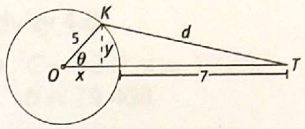
23. The flagpole at Hoover High School stands 25 feet from a 3 foot tall bench. The angle of elevation from the top of the bench to the top of the flag pole is 53°. How tall is the flag pole?

24. From a window on the twelfth story of an apartment building the angle of depression to a playground in Central Park is 22°. If the window is 150 feet above the ground, how far is the playground from the base of the apartment building?

25. An access ramp is being built to retro-fit the local library. The ramp is 30 feet long and 30 inches high at the library entrance. The angle of elevation from the ground to the entrance must be less than 4.8° to pass inspection. Will the ramp pass inspection? Explain.

ACTIVITY 4.5

26. Kyle is riding on a merry-go-round in a counter-clockwise direction. Suppose point K represents his location on the ride as the merry-go-round turns. Kyle's mom Tammy is watching her son at point T. The radius of the merry-go-round is 5 meters and Tammy is 7 meters from the edge of the ride. Kyle's distance d from his mom can be expressed as a function of the angle of rotation θ.



a. What is d when $\theta = 0°$?

b. What is d when $\theta = 150°$?

c. What is d when $\theta = 180°$?

d. What is the range of d?

27. Write the distance d between Kyle and Tammy as a function of angle θ.

28. For one rotation of the wheel, find all values of θ such that d is equal to 10.

29. For one rotation of the wheel, find all values of θ such that d is less than 9.

30. Use the Law of Cosines to solve the following problems.

a. $a = 10, b = 11, C = 45°$. Find c.

b. $b = 9, c = 22, A = 150°$. Find a.

c. $a = 15, b = 62, c = 65$. Find B.

31. What is the measure of the largest angle in a triangle with sides of 3, 5, and 7?

32. Use the Law of Cosines to find the perimeter of a regular pentagon inscribed in a circle with a radius of 6.

13a. $\sin(x + 90°) = \sin x \cos 90° + \cos x \sin 90°$
$= \sin x \cdot 0 + \cos x \cdot 1$
$= \cos x$

18a. $\tan(x + 360°) = \dfrac{\tan x + \tan 360°}{1 - \tan x \tan 360°}$
$= \dfrac{\tan x + 0}{1 - 0}$
$= \tan x$

b. $\sin(x + \pi) = \sin x \cos \pi + \cos x \sin \pi$
$= \sin x \cdot -1 + \cos x \cdot 0$
$= -\sin x$

c. $2\csc 2x = \dfrac{2}{2\sin x \cos x}$
$= \dfrac{1}{\sin x \cos x}$
$= \csc x \sec x$

Scanned by CamScanner

ACTIVITY 4.6

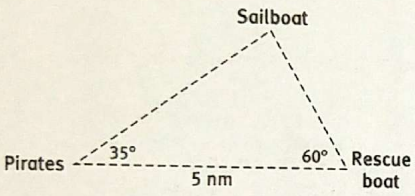
33. Solve each triangle using the Law of Sines.

a. $A = 22°, B = 35°, c = 43$

b. $A = 110°, B = 30°, a = 8$

c. $B = 57°, a = 13, b = 30$

34. A rescue boat and a pirate ship located 5 nautical miles apart both spotted a stranded sailboat at the same time. The rescue boat had a maximum speed of 18 knots (nautical miles per hour) and the pirate ship was capable of 22 knots. The angle between boats is shown below. If both ships set off at their top speed, which one will get to the stranded sailboat first and how long will it take?



35. The angle of elevation from a point 50 yards from a tree to the top of the tree is 23°. The tree leans 4° away from vertical in the direction opposite the point 50 yards away. How tall is the tree?

36. Joaquin is fencing in a triangular pasture. Two posts are located 300 yards apart and the angle from each post to the third one is 75° and 68° respectively. About how much fencing does Joe need?

37. Determine the number of possible triangles for each given situation.

a. $A = 25°, c = 10, a = 5$

b. $B = 63°, c = 90, b = 75$

c. $C = 110°, c = 60, a = 47$

d. $A = 60°, b = 9, a = 9$

38. Solve the two-solution ambiguous case situation.

a. $B = 52°, a = 9, b = 8$

b. $C = 30°, b = 20, c = 12$

UNIT 4 PRACTICE Continued

29. $0° \le \theta < 42.833°$ and $317.167° < \theta \le 360°$

30a. 8.089

b. 30.132

c. 71.894°

31. 120°

32. 35.267

Activity 4.6

33a. $C = 123°, a = 19.207, b = 29.408$

b. $C = 40°, b = 4.257, c = 5.472$

c. $A = 21.311°, C = 101.689°, c = 35.029$

34. Answers may vary. Sample explanation: The rescue boat will arrive first because it takes it 0.1599 hours to get to the sailboat but it takes the pirates 0.1976 hours to get there.

35. 21.926 yds

36. 1243.7 yds

37a. 2 triangles

b. no triangles

c. 1 triangle

d. 1 triangle

38a. $A_1 = 62.438°, C_1 = 65.562°, c_1 = 9.243$ and $A_2 = 117.562°, C_2 = 10.438°, c_2 = 1.839$

b. $B_1 = 56.443°, A_1 = 93.557°, a_1 = 23.954$ and $B_2 = 123.557°, A_2 = 26.443°, a_2 = 10.687$

Scanned by CamScanner