CHAP 7 REVIEW (S-2014)

These exercises are designed to help you review the important ideas and methods of the chapter. Relevant learning objectives are provided in bulleted form before each exercise.

- Distinguish between a parameter and a statistic.

R7.1. **Bad eggs** Sale of eggs that are contaminated with salmonella can cause food poisoning in consumers. A large egg producer takes an SRS of 200 eggs from all the eggs shipped in one day. The laboratory reports that 9 of these eggs had salmonella contamination. Unknown to the producer, 0.1% (one-tenth of 1%) of all eggs shipped had salmonella. Identify the population, the parameter, the sample, and the statistic.

POP = SET OF ALL EGGS SHIPPED ON A GIVEN DAY.

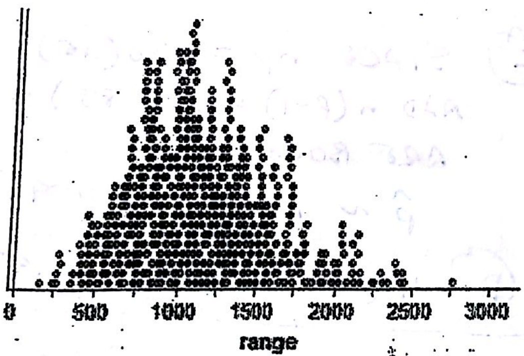
PARAMETER = $p = 0.1\%$ (POP. PROP.)

SAMPLE = SRS OF 200 EGGS

STATISTIC = $\frac{9}{200} = 0.045 = \hat{p}$ (SAM. PROP.)

Exercises R7.2 and R7.3 refer to the following setting. Researchers in Norway analyzed data on the birth weights of 400,000 newborns over a six-year period. The distribution of birth weights is approximately Normal with a mean of 3668 grams and a standard deviation of 511 grams.[9] In this population, the range (maximum − minimum) of birth weights is 3417 grams. We used Fathom software to take 500 SRSs of size $n = 5$ and calculate the range (maximum − minimum) for each sample. The dotplot below shows the results.

$N(3668, 511)$



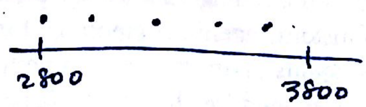
- Understand the definition of a sampling distribution.
- Distinguish between population distribution, sampling distribution, and the distribution of sample data.

R7.2. **Birth weights**

(a) Sketch a possible graph of the distribution of sample data for an SRS of size 5 with a range of 1000 grams.

(b) Explain why the dotplot of sample ranges above is *not* the actual sampling distribution of the sample range.

A



B DOTPLOT DOES NOT GRAPH ALL POSSIBLE SAMPLES OF SIZE 5 THAT CAN BE CHOSEN. IT SHOWS 500 SRS'S.

- Determine whether a statistic is an unbiased estimator of a population parameter.
- Understand the relationship between sample size and the variability of an estimator.

7.3

B) TAKE LARGER SAMPLES TO ↓ VARIABILITY

R7.3. Birth weights

(a) Is the sample range an unbiased estimator of the population range? Give evidence from the graph above to support your answer.

(b) Explain how we could decrease the variability of the sampling distribution of the sample range.

A) NOT UNBIASED ESTIMATOR. IF IT WAS, THEN THE SAMPLING DIST WOULD HAVE RG = 3417. ACCORDING TO GRAPH, NONE OF THE 500 OBS IS > 3000. THE MEAN RG WAS CLOSER TO 1200. SAMPLE RANGE UNDERESTIMATES.

- Find the mean and standard deviation of the sampling distribution of a sample proportion $\hat{p}$ for an SRS of size n from a population having proportion p of successes.
- Check whether the 10% and Normal conditions are met in a given setting.
- Use Normal approximation to calculate probabilities involving $\hat{p}$.

7.4

R7.4. Do you jog? The Gallup Poll once asked a random sample of 1540 adults, "Do you happen to jog?". Suppose that in fact 15% of all adults jog.

(a) What is the mean of the sampling distribution of $\hat{p}$? Justify your answer.

(b) Find the standard deviation of the sampling distribution of $\hat{p}$. Check that the 10% condition is met.

(c) Is the sampling distribution of $\hat{p}$ approximately Normal? Justify your answer.

(d) Find the probability that between 13% and 17% of a random sample of 1540 adults are joggers. Show your work.

A) $\mu_{\hat{p}} = p = 0.15$

B) SINCE ALL ADULTS NUMBER MORE THAN ~~1000~~ $10(1540) = 15400$, WE CAN SAY $\sigma_{\hat{p}} = \sqrt{\frac{0.15(0.85)}{1540}} =$ 0.0091

C) SINCE $np = 1540(.15) = 231$ AND $n(p-1) = 1540(.85) = 1309$, ARE BOTH > 10

$\hat{p} \sim N(0.15, .0091)$

D) $P(.13 < \hat{p} < .17) = \boxed{0.4722}$

- Use the sampling distribution of $\hat{p}$ to evaluate a claim about a population proportion.

R7.5. Bag check Thousands of travelers pass through the airport in Guadalajara, Mexico, each day. Before leaving the airport, each passenger must pass through the Customs inspection area. Customs agents want to be sure that passengers do not bring illegal items into the country. But they do not have time to search every traveler's luggage. Instead, they require each person to press a button. Either a red or a green bulb lights up. If the red light shows, the passenger will be searched by Customs agents. A green light means "go ahead." Customs agents claim that the proportion of all travelers who will be stopped (red light) is 0.30, because the light has probability 0.30 of showing red on any push of the button. To test this claim, a concerned citizen watches a random sample of 100 travelers push the button. Only 20 get a red light.

(a) Assume that the Customs agents' claim is true. Find the probability that the proportion of travelers who get a red light is as small as or smaller than the result in this sample. Show your work.

(b) Based on your results in (a), do you believe the Customs agents' claim? Explain.

A) WE ARE LOOKING FOR PROB THAT IN A SRS OF 100 TRAVELERS 20 OR FEWER GET A RED LIGHT.

$\Rightarrow P(\hat{p} \leq 0.20)$

$\mu_{\hat{p}} = p = 0.3$

PASSENGERS > 10(100) ∴

$\sigma_{\hat{p}} = \sqrt{\frac{(.2)(.8)}{100}} \approx 0.0458$

SINCE $np = 30$ AND $n(p-1) = 70$,

$\hat{p} \sim N(0.3, 0.0458)$

$P(\hat{p} \leq .20) = \boxed{0.0146}$

B) CLAIM UNLIKELY TO BE TRUE. THERE IS ONLY A 1.5% CHANCE THAT WE WOULD FIND A SAMPLE W/THAT FEW RED LIGHTS.

- Find the mean and standard deviation of the sampling distribution of a sample mean $\bar{x}$ from an SRS of size *n*.
- Calculate probabilities involving a sample mean $\bar{x}$ when the population distribution is Normal.
- Explain how the shape of the sampling distribution of $\bar{x}$ is related to the shape of the population distribution.

R7.6. IQ tests The Wechsler Adult Intelligence Scale (WAIS) is a common "IQ test" for adults. The distribution of WAIS scores for persons over 16 years of age is approximately Normal with mean 100 and standard deviation 15.

(a) What is the probability that a randomly chosen individual has a WAIS score of 105 or higher? Show your work.

(b) Find the mean and standard deviation of the sampling distribution of the average WAIS score $\bar{x}$ for an SRS of 60 people.

(c) What is the probability that the average WAIS score of an SRS of 60 people is 105 or higher? Show your work.

(d) Would your answers to any of parts (a), (b), or (c) be affected if the distribution of WAIS scores in the adult population were distinctly non-Normal? Explain.

A) $N(100, 15)$

$P(W \geq 105) = \boxed{0.3707}$

B) $\mu_{\bar{x}} = \boxed{100}$. $\sigma_{\bar{x}} = \frac{15}{\sqrt{60}} = \boxed{1.9365}$

($\bar{x} \sim$ NORMAL)

C) $P(\bar{x} \geq 105) = \boxed{0.0049}$

D) A) COULD BE VERY DIFFERENT.
B) WOULD BE SAME SINCE $\mu_{\bar{x}}$ AND $\sigma_{\bar{x}}$ DO NOT DEPEND ON SHAPE OF DIST.
C) STILL PROBABLY RELIABLE BECAUSE 60 > 30 (CLT).

Use the central limit theorem to help find probabilities involving a sample mean $\bar{x}$.

R7.7. Detecting gypsy moths The gypsy moth is a serious threat to oak and aspen trees. A state agriculture department places traps throughout the state to detect the moths. When traps are checked periodically, the mean number of moths trapped is only 0.5, but some traps have several moths. The distribution of moth counts is discrete and strongly skewed, with standard deviation 0.7.

(a) What are the mean and standard deviation of the average number of moths $\bar{x}$ in 50 traps?

(b) Use the central limit theorem to help you find the probability that the average number of moths in 50 traps is greater than 0.6.

A) $\mu_{\bar{x}} = 0.5$

$\sigma_{\bar{x}} = \frac{0.7}{\sqrt{50}} = 0.0990$

B) ASSUME $n > 30$

$P(\bar{x} > 0.6)$ FROM $N(0.5, 0.099$

$= \boxed{0.1562}$