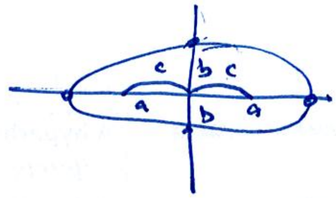
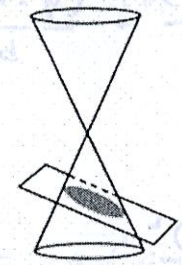


$c^2 = a^2 - b^2$

Activity 5.2 – Ellipses and Hyperbolas

MATH TERMS

Conic sections are formed by the intersection of a plane and a double-napped cone. The figure below shows a plane intersecting a cone in an ellipse.



An **ellipse** is the set of all points (x, y) in a plane such that the *sum* of the distances from (x, y) to each of two fixed points is a constant. Each of the fixed points is called a **focus** of the ellipse.

Standard Form of the Equation of an Ellipse

The standard form of the ellipse with center (h, k), major axis length 2a and minor axes of length 2b, where 0 < b < a is given by:

Horizontal major axis:

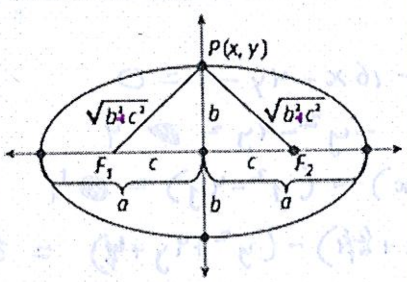
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Vertical major axis:

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

The distance between the foci is **2c**, where $\mathbf{c^2 = a^2 - b^2}$.

MATH TIP



The major axis of the ellipse has length $2a$, the minor axis has length $2b$ and the distance between the two foci is $2c$.

Since the point $P(x, y)$ is a point on the ellipse,

$F_1P + F_2P = 2a$

$\sqrt{b^2+c^2} + \sqrt{b^2+c^2} = 2a$

$2\sqrt{b^2+c^2} = 2a$

$\sqrt{b^2+c^2} = a$

$b^2 + c^2 = a^2$.

Therefore $c^2 = a^2 - b^2$.

Examples: P. 312

ex 6a. $9x^2 + 4y^2 + 36x - 24y + 36 = 0$

$9x^2 + 36x \quad + 4y^2 - 24y = -36$

$9(x^2 + 4x) + 4(y^2 - 6y) = -36$

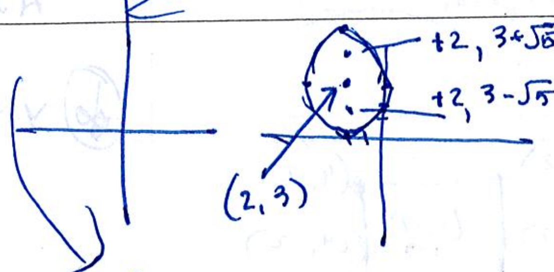
$9(x^2 - 4x + 4) + 4(y^2 - 6y + 9) = -36 + 36 + 36$

$9(x^2 - 4x + 4) + 4(y^2 - 6y + 9) = 36$

$\cancel{x^2 - 4x + 4} + \frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$

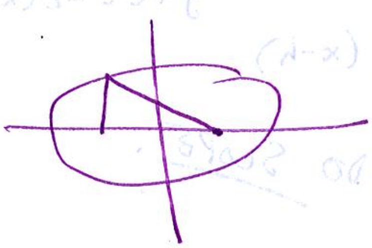
$\cancel{c^2 = 9^2 - 4^2 = 81 - 16}$

$c^2 = 9 - 4 \Rightarrow c = \sqrt{5}$



$2a = 2(3) = 6$

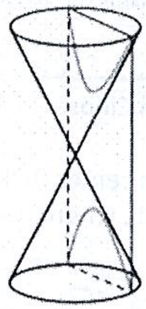
$2b = 2(2) = 4$



Do ex. 6b

MATH TERMS

The figure below shows a plane intersecting a cone in a hyperbola.



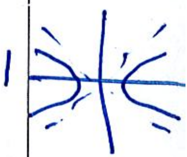
A **hyperbola** is defined as the set of points (x, y) such that the *difference* of the distance from (x, y) to each of the foci is constant.

Standard Form of the Equation of a Hyperbola

The standard form of the equation of a hyperbola with center (h, k) and transverse axis of length of 2a is given by:

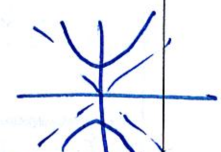
Horizontal transverse axis: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

LOOKS LIKE AN "X"



Vertical transverse axis:

LOOKS LIKE A "Y". $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$



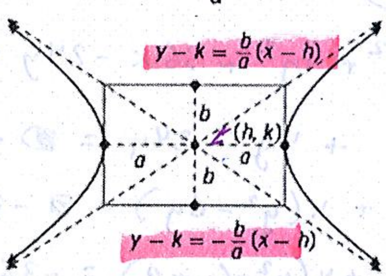
The distance between the foci is **2c**, where $c^2 = a^2 + b^2$. The foci are located at **(h +/- c, k) and (h, k +/- c)**.

MATH TIP

The asymptotes of a hyperbola contain the center of the hyperbola and contain the vertices of a rectangle with dimensions 2*a* and 2*b*.

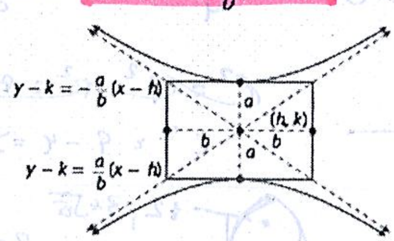
The equations of the asymptotes of a hyperbola with a horizontal transverse axis are

$$y - k = \pm\frac{b}{a}(x-h)$$



The equations of the asymptotes of a hyperbola with a vertical transverse axis are

$$y - k = \pm\frac{a}{b}(x-h)$$



Examples:

(11a) $4x^2 - y^2 - 16x - 4y - 4 = 0$

$4x^2 - 16x \; - y^2 - 4y = 4$

$4(x^2 - 4x) - (y^2 - 4y) = 4$ $\quad 4 + 16 - 4$

$4(x^2 - 4x + 4) - (y^2 + 4y + 4) = 16$

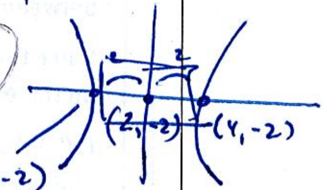
$4(x-2)^2 - (y+2)^2 = 16$

(BOX) $\frac{(x-2)^2}{4} - \frac{(y+2)^2}{16} = 1$

$C(2,-2)$ BOX

$V_1(4,-2)$ $V_2(0,-2)$

$F_1(2-2\sqrt{5},-2)$ $F_2(2+2\sqrt{5},-2)$



$c^2 = 4 + 16 = 20 \Rightarrow c = 2\sqrt{5}$

ASYMPTOTES. $y+2 = 2(x-2)$

$y+2 = -2(x-2)$

(b) $y - k = \pm\frac{b}{a}(x-h)$

DO SLOPE.